Eurasian Journal of Educational Research 78 (2018) 117-138

Eurasian Journal of Educational Research
www.ejer.com.tr

# The Effect of Teaching Integers through the Problem Posing Approach on Students' Academic Achievement and Mathematics Attitudes*

Ahmet Sukru OZDEMIR[1] , Muhammet SAHAL[2]

ARTICLE INFO

*Article History:*
Received: 04 Jul. 2017
Received in revised form: 11 Sep. 2018
Accepted: 01 Nov. 2018
DOI: 10.14689/ejer.2018.78.6

***Keywords***
mathematics education, problem posing, problem solving, teaching integers

ABSTRACT

**Problem Statement:** Throughout history, many changes have occurred in the field of mathematics education. These changes have also occurred concerning special topics that mathematics educators have constantly been searching. The significance of problem posing in mathematics teaching has increased recently with respect to its contributions to the teacher and the student. Thus, the problem posing approach is examined with respect to special topics. **Study Purpose:** The effect of teaching integers through the problem posing approach on sixth grade students' academic achievement and mathematics attitudes.

**Method:** Mixed method, in which quantitative and qualitative research methods are used together, was conducted in the study. While the pre-test post-test control group model constituted the quantitative dimension of the study, the observation method and content analysis of students' work sheets were used for the qualitative dimension. The study groups consisted of a total of 69 participants, 34 of them were in the experimental and 35 of them were in the control group. **Findings and Results:** According to the findings of the study, there was a difference in favor of the experimental group with respect to the academic achievement levels; and there were no significant differences between two groups with respect to the attitudes towards mathematics. The observations indicated that the problem posing approach created a peaceful competition environment, and increased participation in the classroom. In addition, the student work sheets showed that participants' problem posing skills progressed, and they became aware of their mistakes. In conclusion, the problem posing approach had a positive effect on the academic achievement in teaching integers, but it did not have a significant effect on student attitudes towards mathematics.



*This study is based on the master's thesis prepared by Muhammet Sahal with the same title under supervision of Ahmet Sukru Ozdemir and was partly presented at the 3rd International Eurasian Educational Research Congress in Mugla, 31 May – 03 June, 2016.

[1] Marmara University, TURKEY, e-mail: ahmet.ozdemir@marmara.edu.tr
ORCID: https://orcid.org/0000-0002-0597-3093

[2] Yildiz Technical University, TURKEY, e-mail: msahal@yildiz.edu.tr,
ORCID: https://orcid.org/0000-0003-3625-2456

## Introduction

The constructivist learning approach has been adopted in Turkey since 2006. The mathematics program, which was updated in 2013, is being implemented on the secondary school level. This reform movement positively affected mathematics education which was in a constant progress (Baki, 2015). Along with this, other progresses were made, but none of these progresses have been considered sufficient.

Turkey also participates in international exams. According to Program of International Student Assessment (PISA) results, which were carried out in Turkey in 2006, males' score for mathematics performance was 427 while females' score was 421 (OECD, 2006). While males' score was 451 in the exam in 2009, females' score was 441 (OECD, 2009). In the exam carried out in 2012, males' score for mathematics performance was 452 while females' score was 444 (Organisation for Economic Co-operation and Development [OECD], 2012). According to the records, these scores remain below the average of OECD.

One of the biggest role in achieving to attain program objectives is undoubtedly assigned to teachers. It can be stated that mathematics educators have a big role in shaping the future. Ersoy (2005) stated that mathematics has been the primary determinant of the progress and developments in science and technology throughout the previous century. Thus, it is crucial for teachers to adapt to life which is constantly undergoing a change. With respect to educational program objectives, teachers should question, be open to self-development, understand the value of mathematics and recognize it, master the relationships between concepts, and should aim at raising students who learn how to learn.

Preparing students to real life through improving their problem solving skills is one of the objectives of our educational system. In the program, problem solving was found important in every stage of secondary school mathematics education, and it was stated that this skill should be improved (Ministry of National Education [MoNE], 2013). Teaching through problems helps students to internalize the concepts and improve their skills (Akay, Soybas & Argun, 2006). A problem is a state that includes open questions, attracts one's attention to questions which the individual does not have the experience and knowledge to answer (Blum & Niss, 1991). The student should be interested in the problem, be motivated to solve it, and the problem should intrigue the student.

The student's problem solving skill is expected to improve through the efforts made for the problems. According to Polya (1957), solving a problem refers to examining the solution for obtaining what is openly thought. Solving a problem is the process and method of using knowledge together with a new and unordinary (routine) solution. Polya's four step method is categorized as understanding the problem, devising a plan, carrying out the plan, and looking back (Polya, 1957). Polya's problem solving method is commonly adopted to the mathematical problem solving process.

The tendency towards problem posing has increased recently along with problem solving. Problem posing is an action which is defined as creating a new problem or reformulating a problem (Silver, 1994). Problem posing is considered as the fifth stage of the four step process introduced by Polya (Gonzales, 1998). National Council of Teachers of Mathematics (2000) refers to problem posing as an alternative approach in contemporary educational approach. Problem posing is crucial due to its contributions to problem solving skills and the attitudes towards mathematics (Silver, 1994; Turhan & Guven, 2014). Negative attitudes towards mathematics arise when the problem is not understood, when a plan is not found about it, and when the solution is not attained (Cankoy & Darbaz, 2010). The problem posing approach can be used to overcome this situation. As Akay and Boz (2009) state, problem posing provides endearing mathematics, improves problem solving skills, ensures conceptual understanding, and supports democratic learning environment. Akay (2006) states that a lesson taught through the problem posing approach positively affects the academic achievement levels and problem solving skills of students. Lowrie (1999) asserts that giving students the opportunity to create their own problems enables them to observe their beliefs and attitudes towards mathematics. Studies show that problem posing can decrease common fears and concerns towards mathematics (English, 1997; Silver, 1994). With this respect, problem posing can be considered as crucial.

Considering problem posing separate from problem solving is equal to ignoring most of the features of problem posing. According to Baki (2015), any condition that disturbs an individual or which the individual cannot predict the solution for is defined as a problem. An issue is referred to as a problem when it disturbs us, and when we don’t know its solution beforehand. Thus, Kontorovicha, Koichua, Leikin and Bermana (2012) state that a problem that needs to be posed is also a problem that is waiting to be solved, and they emphasize that problem posing is a special way of problem solving. A good problem solver is never done after having reached the solution. When the solution of a problem is found, another way should be sought, and other ways of solving it should be tried (MoNE, 2013). The existence of other solutions should be studied and discussed. Lavy and Shiriki (2010) state that problem posing helps students change their strict opinion that a problem only has a unique solution; and thus, it is related to problem solving. One kind of problem posing referred to as problem re-formulation occurs within the process of problem solving (Silver, 1994). This method referred to as “Working Backward” involves the type of problem posing by reformulating an existing problem (Polya, 1957). Because when the individual does not know how to solve a problem with this strategy, then he or she tries to create another simpler problem similar to the previous one. The relationship between problem posing and problem solving was emphasized by many researchers in the literature (Akay & Boz, 2009; Cankoy & Darbaz, 2010; Ellerton, 1986; English, 1997; Kar, Ozdemir, Ipek & Albayrak, 2010; Kontorovicha, Koichua, Leikin & Bermana, 2012; Lavy & Shiriki, 2010; Lowrie, 2002; Unlu & Sarpkaya Aktas, 2017). Problem posing is also crucial due to its relationship with problem solving.

Integers can be considered as one of the subjects that includes relationship between problem posing and problem solving. It constitutes a large proportion in the teaching program. Students in primary school are introduced with natural numbers and the positive line of rational numbers. Introduction to integers in secondary school period constitutes a problem for many students. When we consider the literature, it is obvious that the integers is one of the subjects that has been widely studied (Aydin Unal & Ipek, 2009; Bingolbali & Ozmantar, 2014; Ercan, 2010; Isguden, 2008; Koroglu & Yesildere, 2004; Korukcu, 2008; Varol & Kubanc, 2012). However, it is still one of the subjects that is found mostly difficult in mathematics education (Yenilmez & Bagdat, 2014). In addition, difficulties, conceptual errors and mistakes on this subject can lead to several problems in teaching of the future subjects. Because the integers is a prominent subject for many following topics. Thus, integers is considered as a crucial subject in the secondary school mathematics curriculum. It is evident that previous studies on problem posing have focused mostly on conceptual mistakes and student difficulties (Akay & Boz, 2009; Cankoy & Darbaz, 2010; Kar, Ozdemir, Ipek & Albayrak, 2010; Turhan & Guven, 2014). The variety and depth of studies should be expanded to subjects that students find difficult. Thus, this study is expected to contribute to researchers and teachers in teaching of the integers.

### *Study Purpose*

It was observed that studies on problem posing focused on the characteristics of the posed problems, and attitudes and problem posing skills of teachers and preservice teachers. There are limited studies focusing on the use of problem posing activities in teaching special subjects. Particularly, studies with classroom activities in which problem posing is used in secondary school are limited. Therefore, sixthgrade students were chosen for the study. The effects of the problem posing approach on teaching the integers was the concern of this study. The purpose of the study is to examine whether any significant difference occurs in sixth grade students' academic achievement levels regarding integers and their attitudes toward mathematics after teaching the integers through the problem posing approach. With this respect, this study is thought to contribute to the related literature. Based on this aim, the following research questions were asked:

1. Is there a significant difference in sixth garde students' academic achievement levels between the experimental and control groups after teaching the integers through the problem posing approach?

2. Is there a significant difference in sixth garde students' attitude towards mathematics between the experimental and control groups after teaching the integers through the problem posing approach?

3. What kind of problems were posed by sixth grade students while learning with problem posing activities?

## Method

### *Study Design*

Explanatory design, which is a type of mixed method, was used in the current study because it is used when a researcher also needs qualitative data in addition to quantitative data in order to explain significant results (Morse, 1991). The "pre-test/post-test control group" research design, which is a type of quasi-experimental designs, was conducted for the quantitative dimension of the study. In this research model, the data are observed under the control of the researcher, and it helps to determine the cause-effect relationship (Karasar, 2014, p. 87).

Observation and document analysis were conducted for the qualitative dimension of the study. Content analysis is a scientific approach which enables examining verbal, written and other materials in an objective and systematical way (Tavsancil & Aslan, 2001). The unstructured field notes were used in the study. In this type of observation, the researcher enters the natural setting and tries to carry out a "participant observation" by becoming a member of the setting, and the researcher may not need any standard observation or interview instruments (Yildirim & Simsek, 2013).

### *Population and Sample*

The study population consisted of sixth grade students, who were from low socio-economic regions, studying in Istanbul. The study group of the study consisted of 6/B and 6/C students studying at a secondary school. A total of 69 students participated in the study. The students who participated in the study were selected through the random assignment method. Random assignment method is a technique for assigning participants to different groups in a study by using randomization (Karasar, 2006). The random assignment method was used in the current study because it is proper for the quasi-experimental design with pre-test post-test control group (Buyukozturk, 2014).

### *Data Collection Tools*

The "Mathematics Attitude Scale", developed by Erktin and Nazlicicek (2002), was used in the study to determine the attitudes of the experimental and control groups towards mathematics. The scale involves 20 items, and a maximum of 100 points can be taken. The Cronbach's Alpha reliability coefficient of the scale was 0.735. In addition, the Integers Subject Achievement Test was also used in the study. It consists of 21 items, and it was applied before and after the implementation to determine the students' achievement levels. The period between the first and the second implementation was five weeks. The achievement test was developed by a lecturer, the researcher of the study and three expert teachers, who were a linguist, a mathematics teacher and an assistant professor. They reviewed the scale according to the acquisition process of the integers, and then a pilot test was carried out. The achievement test consists of 21 multiple choice questions. A maximum of 21 points can be taken from the test. The KR 21 reliability coefficient of the Integers Subject Achievement Test after the implementation was calculated as 0.717. Lastly, the Problem Posing Evaluation Rubric, developed by Katranci (2014), was used to categorize the problems posed by the students. The rubric consists of 4 dimensions. These dimensions are language and expression (1), mathematical adaptation of the problem (2), structure/type of the problem (3), and solvability of the problem (4).

The implementation was carried out with the lesson plans prepared according to the problem posing approach, and it was ensured that every component of problems were produced by students throughout the study students. After asking the students to create problems similar to the examples and problems in their daily lesson plan, they were told to pose a problem based on a story or a piece of information, and they were told to solve the problem as their homework. For example, students were asked to propose a problem about weather condition by using real data. If the problem posed by the student had not been valid mathematically, or it had logical errors or it had been too simple for the level of the class, then the problem would have been changed by the students. The problems and examples posed were solved in the classroom setting.

While the integers was taught the control group by following the approach mentioned in the curriculum, problem posing approach was used in the experimental group. The same teacher taught both groups. The study lasted for five weeks. Instruction included objectives given in the program such as sorting out integers, four modes of operations, and number line. The sources of the examples and problems in the lessons of the control group were subsidiary sources or the teacher.

*Data Analysis*

The data obtained during 2015-2016 educational term were analyzed through the SPSS 17 software. The Kolmogorov Smirnov test was carried out for the normality test. The independent samples t-test, paired samples t-test, Wilcoxon and Mann Whitney U Test were conducted in terms of statistical analyses. The observation notes were gathered during the implementation. The data gathered through student work sheets were analyzed through the document analysis method. The rubric was used to analyse problems posed by the students.

The Kolmogorov-Smirnov Test was conducted on the Integers Subject Achievement Test, and the results showed that the data had normal distribution.

Similarly, the Kolmogorov-Smirnov Test was also conducted on the Mathematics Attitude Scale, and the results showed that while the pre-test results of the experimental and control groups had normal distribution, the post-test results did not have a normal distribution.

## Results

*Results Concerning the Achievement Test*

Results of the independent sample t-test concerning the experimental and control groups' achievement test scores were given in Table 1.

**Table 1**
*Results of the Independent Sample t-Test Concerning the Experimental and Control Groups' Achievement Test Scores*

| Group/Test | N | $\overline{X}$ | sd | t | p |
|---|---|---|---|---|---|
| Experimental Group Pre-test | 34 | 9.12 | 2.868 | 0.653 | 0.516 |
| Control Group Pre-test | 35 | 8.66 | 2.990 | | |
| Experimental Group Post-test | 34 | 12.85 | 4.024 | 2.175 | 0.033 |
| Control Group Post-test | 35 | 10.80 | 3.818 | | |

According to Table 1, there were no significant differences between the experimental and control groups before the implementation at the 0.05 significance level about the topic of integers. It can be said that both experimental and control groups were equal at the beginning of the study. When the post-test results of the achievement test were considered, there was a difference at the 0.05 level in favor of the experimental group. It was evident that the experimental group's average was 12.85, and the control group's average was 10.80. Results of the paired sample t-test concerning the experimental and control groups' achievement test scores were given in Table 2.

**Table 2**

*Results of the Paired Sample t-Test Concerning the Experimental and Control Groups' Achievement Test Scores*

| Group/Test | N | $\overline{X}$ | sd | t | p | r | Cohen's d |
|---|---|---|---|---|---|---|---|
| Experimental Group Pre-test | 34 | 9.12 | 2.868 | -8.168 | 0.000 | 0.750 | |
| Experimental Group Post-test | 34 | 12.85 | 4.024 | | | | |
| Control Group Pre-test | 35 | 8.66 | 2.990 | -4.415 | 0.000 | 0.669 | 0,530 |
| Control Group Post-test | 35 | 10.80 | 3.818 | | | | |

According to Table 2, there was a significant difference between pre-test and post-test scores of the experimental group students with respect to the achievement test. When the arithmetical averages of the groups were considered, it was evident that the post-test scores (12.85) were higher than the pre-test scores (9.12). When the pre-test and post-test scores related to the integers numbers subject achievement test of the control group students were considered, it was evident that there was a significant difference between their pre-test and post-test scores. When the arithmetical averages of the groups were considered, it was evident that the post-test scores (10.80) were higher than the pre-test scores (8.66). Also when Cohen's d was considered (0,530), it can be said that the effect size of problem posing activities was at medium level (Cohen, 1994).

### *Results on the Mathematics Attitudes Scale*

The Mathematics attitude scale ındependent samples t-test results of the experimental and control groups before the ımplementation were given in Table 3.

**Table 3**

*The Mathematics Attitude Scale Independent Samples t-test Results of the Experimental and Control Groups before the Implementation*

| Group/Test | N | $\overline{X}$ | sd | t | p |
|---|---|---|---|---|---|
| Experimental Group | 34 | 79.50 | 11.296 | 1.373 | 0.174 |
| Control Group | 35 | 76.06 | 9.487 | | |

According to Table 3, there were no significant differences between the experimental and control groups' mathematic attitudes at the 0.05 significance level before the implementation. The Mathematics Attitude Scale Mann Whitney U test results of the experimental and control groups after the ımplementation were given in Table 4.

**Table 4**

*The Mathematics Attitude Scale Mann Whitney U Test Results of the Experimental and Control Groups after the Implementation*

| Group/Test | N | $\overline{X}$ | sd | z | p |
|---|---|---|---|---|---|
| Experimental Group | 34 | 81.50 | 11.035 | -1.562 | 0.118 |
| Control Group | 35 | 78.37 | 10.502 | | |

According to Table 4, there were no significant differences in the mathematics attitudes of the experimental and control group students after the implementation. The Mathematics Attitude Scale Wilcoxon Signed-Ranks test results of the experimental group were given in Table 5.

**Table 5**

*The Mathematics Attitude Scale Wilcoxon Signed-Ranks Test Results of the Experimental Group*

| Group/Test | N | $\overline{X}$ | sd | z | p | r |
|---|---|---|---|---|---|---|
| Experimental Group Pre-test | 34 | 79.50 | 11.296 | -2.054 | 0.040 | 0.750 |
| Experimental Group Post-test | 34 | 81.50 | 11.035 | | | |

When Table 5 was taken into consideration, it was evident that there was a significant difference between the Mathematics Attitude Scale pre-test and post-test score averages after the implementation in favor of the post-test scores. It was evident that the post-test scores (81.50) were higher than the pre-test scores (79.50).

**Table 6**

*The Mathematics Attitude Scale Wilcoxon Signed-Ranks Test Results of the Control Group*

| Group/Test | N | $\overline{X}$ | sd | z | p | r |
|---|---|---|---|---|---|---|
| Control Group Pre-test | 35 | 76.06 | 9.487 | -1.866 | 0.062 | 0.399 |
| Control Group Post-test | 35 | 78.37 | 10.502 | | | |

According to the results on Table 6, there were no significant differences between the Mathematics Attitudes Scale pre-test and post-test scores of the control group. Because the subjects were taught to the control group students as stated in the curriculum, and no additions were made in the way of teaching the subjects, it can be interpreted that no significant differences occurred in the control group students' mathematics attitudes.

### *Results Concerning the Observation and Work Sheets*

Results Concerning the Work Sheets were given in Table 7.

**Table 7**

*Results Concerning the Work Sheets*

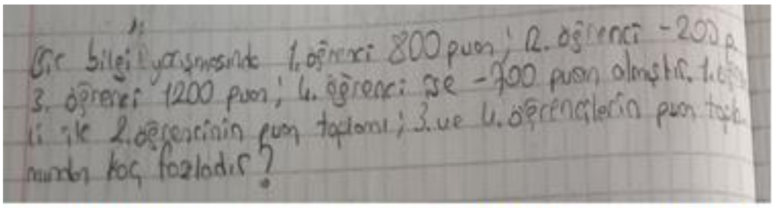
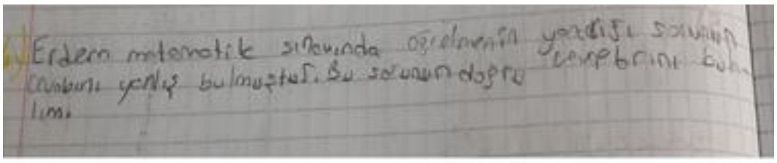
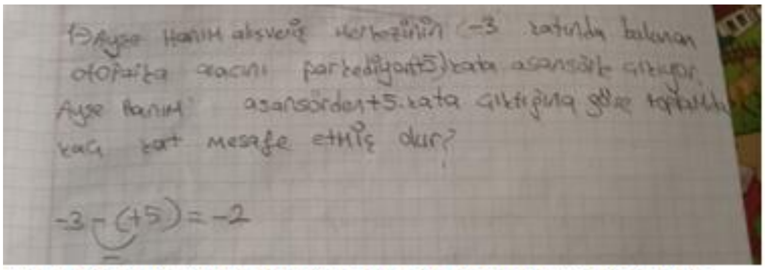
| Student | Posed Problem |
|---|---|
| S25 |  In a quiz show, the score of the first student is 800; the second student is -200; the third student is 1200; the fourth student is -700. How much more is the sum of scores of first and second students than the sum of the scores of the third and fourth students? |
| S4 |  Erdem found the answer of the question that the teacher asked in the mathematics exam wrong. Let's find the correct answer of this question. |
| S7 |  Mrs. Ayşe parked her car in the carpark area in the (-3) floor of a shopping center. She goes to (+5) floor with the elevator. If Mrs. Ayşe went to the $+5^{th}$ floor of the shopping center, how many floors did she go up? |

Table 7 Continue

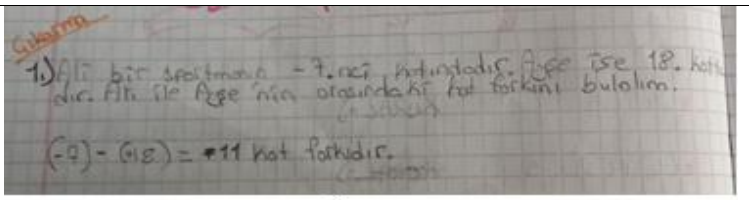
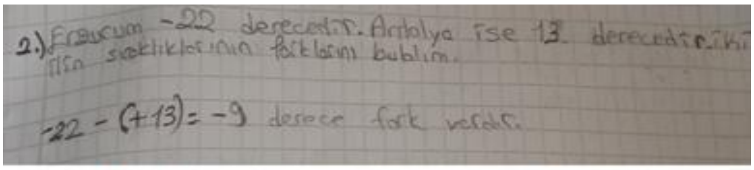
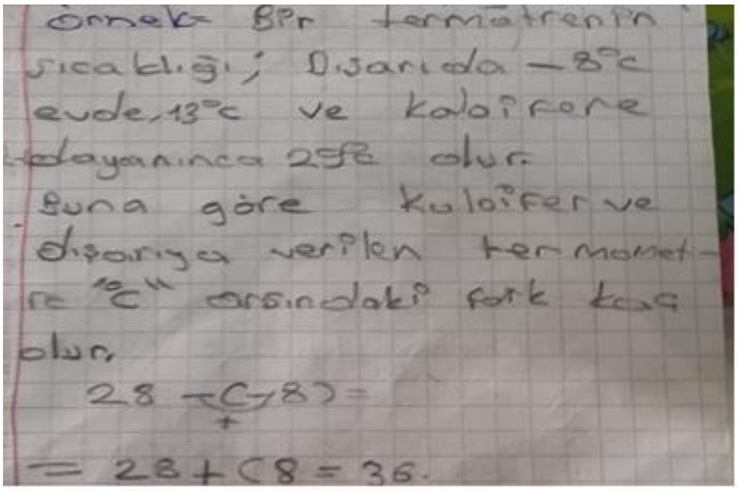
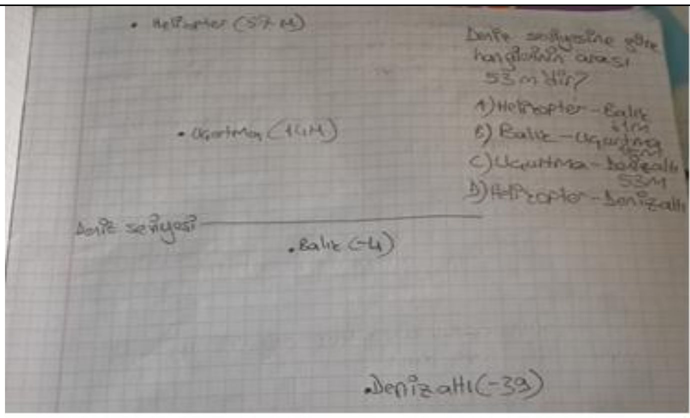
| Student | Posed Problem |
|---|---|
| S21 | <br>Subtraction 1.) Ali is at the -7th floor of an apartment. Aye is at the 18th floor. Let's find the floor gap between Ali and Ayşe.<br>(-7) – (+18) = * 11 floors |
| S17 | <br>It is -22 ° C at Erzurum. It is 13 ° C at Antalya. Let's find the temperature gap between the two cities.<br>-22 - (+13) = -9 celsius degress gap |
| S24 | <br>Example= The temperature of a thermometer is; -8°C outside, 13°C at home and 29°C when you lean on the radiator. What is the "°C" difference between the radiator and the temperature on the thermometer put outside? |

Table 7 Continue

| Student | Posed Problem |
|---|---|
| S22 | <br>•Helicopter (57 m)<br>•Kite (14 m)<br>Sea Level<br>•Fish (-4)<br>•Submarine (-39)<br>According to the sea level, between which two is 53 m.?<br>A) Helicopter – Fish 61m<br>B) Fish – Kite 18m<br>C) Kite – Submarine 53m<br>D) Helicopter – Submarine |

It was evident that the language of the problem posed by S25 was open and clear. The problem was solved by the class easily. During the solution of the problem in the classroom, the student was asked by his classmates how a negative score was obtained. The student stated that incorrectly answered questions could result in negative scores; and thus, he gave a word to a classmate and made him or her solve the problem. It was evident that the problem was a verbal type of problem.

It was evident that the problem posed by student number 4 had a problem, but it lacked the other components; and thus, it was not resolvable because there was no question in the problem produced by the student. Classmates asked to the students where 'the problem' is. Then the teacher interfered and asked the students to re-create the questions that were asked incorrectly.

The questions posed by S7, S21 and S17 were observed to have similar mistakes. The gap between a positive number and negative number was mixed up with the gap between two positive numbers; and this was revealed when the posed problems were being resolved. It was evident that despite small deficiencies, the problems were comprehensible, mathematically resolvable, and were compatible with real life conditions with respect to the rubric. These mistakes were discussed in the classroom and corrected.

Despite small deficiencies, the problem posed by S24 could be characterized as comprehensible, resolvable, and related to real life conditions. It was also evident that the student solved the posed problem correctly. The student was able to comprehend the difference between positive and negative numbers.

The problem posed by S22 could be interpreted as open, clear and comprehensible. The problem was mathematically valid. Since the result of the problem was asked, all the related choices were considered in the classroom, and a different student was given the floor to speak for each choice, the learning of integers was supported.

It was observed that there was a friendly competition in the classroom among the students about asking the problem they posed to their classmates. The teacher stated that participation slightly increased in the classroom. When the given examples were considered, it was obvious that student mistakes were revealed.

Nineteen students (54,28%) in the experimental group posed problems through problem posing activities. Total number of problems posed by 19 students was 76. These problems were classified with respect to the rubric. Whereas 51 of the problems (67.10%) were found as understandable, clear and fluent, 25 of them (32,89%) were found as mathematically expressed. Furthermore, 19 of 76 problems (25%) were found as solvable, but other 57 questions were not resolvable due to logical errors, unclear expressions and inconsistent knowledge.

**Table 8**

*Assessment of problems posed by students with respect to the rubric*

| | f | % |
|---|---|---|
| Languange and expression | 51 | 67,10 |
| Mathematical adaptation | 25 | 32,89 |
| Solvability | 19 | 25 |

## Discussion, Conclusion and Recommendations

The purpose of this study was to examine the effect of the problem posing approach on the sixth grade the students' academic achievement and attitudes toward mathematics.

It was evident that the experimental and control groups were equal to each other before the implementation. It was revealed that there was a significant difference between the control groups' pre-test and post-test achievement scores in favor of the post-test scores. The educational activities could be said to have a positive effect on the academic achievement levels of the students. Because of the fact that both of the groups were exposed to teaching, they all increased their achievement scores. Tekin (1996) also explains this situation that no matter which approaches are adopted in the educational practices, the eventual aim is to attain desired behaviors. According to the experimental group's pre-test and post-test data, there was a significant difference in favor of the post-test scores. In addition, there was a significant difference in favor of the experimental group with respect to the integers achievement levels after the implementation. It was revealed that experimental group's post-test scores were higher than control group's post-test scores (see Table 1). Thus, it could be interpreted that the problem posing approach had a positive effect on the experimental groups' success in learning the integers. With respect to these findings, it could be stated that the problem posing approach affected academic achievement on learning the integers. There are many studies in literature (Akay, 2006; Akay and Boz, 2009; Arikan and Unal, 2013; Fidan, 2008, Unlu and Sarpkaya Aktas, 2017) which state that problem posing activities increase academic achievement of students. Turhan and Guven (2014) and Abu-Elwan (1999) underline that the problem posing approach improves the problem solving skills of students. Lowrie (1999) observed that problem posing activities within the classroom enable a deeper understanding of the problem solving processes for the students. Ghasempuor, Bakar and Jahanshahloo (2013) suggest recreating problem posing in science classes because it facilitates learning. Katranci (2014) states that problem posing increases students' understanding in mathematics. Findings of the study support these studies.

Mathematics attitude scale scores of the experimental and control groups before the implementation could be considered equal. It was observed that there were no significant differences in the mathematics attitude scale post-test scores of the experimental and control groups after the implementation. When the findings were considered, it could be stated that the teaching of integers through the problem posing approach did not have a significant effect on the mathematics attitudes of the students. These results contradict with the results of the studies conducted by Bakar and Jahanshahloo (2013), Ghasempour, Karasel, Ayda and Tezer (2009), Lowrie (1999), Silver (1994), Silver and Cai (1996) and Turhan and Guven (2014). These studies propose that problem posing has a positive effect in student attitudes towards mathematics. An attitude is a positive or negative manner, or an evaluation of expression about objects, people, events or a state (Karakas Turker & Turanli, 2008; Ustuner, 2006). In addition, the researchers also emphasize that the attitudes do not change in a short period of time (Koballa, 1988; Narrated by: Turgut & Gurbuz, 2011; Sahin, Ongoren & Cokadar, 2010). Because the integers was taught through the problem posing approach in a short period of 3 weeks, it was considered that there were no changes in the attitudes of the students. The duration of teaching in the stated studies was minimum 8 weeks long. The reason for this can be because a long period of time is necessary to change attitudes.

According to the observations, it was evident that the students were willing to participate in the lesson during the problem posing activities, and that there was a friendly competition among the students about asking the problem they posed to their classmates. The mistakes of the students were revealed through the problems they posed (see S17's on Table 7). Thus, the students got the opportunity to think once again about the mathematical concepts they had acquired. In addition, because the posed problems were re-evaluated by classmates, this gave the opportunity to correct the mistakes. These results are in line with the studies conducted by English (1997) and Lowrie (1999), Lavy and Shiriki (2010), Toluk Ucar (2009). These studies also assert that problem posing activities can be used as evaluation instruments for revealing the conceptual mistakes and errors of the students. In addition, 19 of 35 students in the experimental group posed 76 problems. One quarter of these problems were appropriate in terms of language and expression, they were solveable and mathematically valid. More than half of the experimental group participated in the activities actively. The findings of the current study are in line with the other ones such as Arikan and Unal (2013), Cankoy and Darbaz (2010), and Lowrie (1999; 2002).

According to the results of the study, problem posing can be suggested to mathematics teachers for classroom and out-of-classroom activities that have an appropriate structure. When the benefits of problem posing activities are considered, they should be included more widely in the curriculum. Teachers should use the problem posing approach as an alternative teaching and evaluation instrument. Thus, they will get the opportunity to understand the deficient learning experiences of students, their conceptual mistakes and their full knowledge of the subject. This approach can be used more frequently through additional lessons. Furthermore, this approach can be used not only in mathematics but also in other courses such as physics and chemistry. The study is limited only to sixth grade students and to the topic of integers. A more comprehensible study can be carried out on the effects of problem posing in other levels and courses. In order to acknowledge teachers and pre-service teachers, trainings should be provided to create awareness on the importance and implementation of problem posing.

## References

Abu-Elwan, R. (1999). The development of mathematical problem posing skills for prospective middle school teachers. In A. Rogerson (Ed.), *Proceedings of the ınternational conference on mathematical education into 21st Century: Social challenges, ıssues and approaches (pp. 1-8). Cairo: Egypt.*

Akay, H. (2006). *Analysis of the effects of mathematics education carried out through the problem posing approach on the academic achievement, problem solving skills and creativity of students* (Doctorate dissertation). Accessed on https://tez.yok.gov.tr

Akay, H., & Boz, N. (2009). Prospective teachers' views about problem-posing activities. *Procedia Social and Behavioral Sciences, 1*, 1192-1198.

Akay, H., Soybas, D., & Argun, Z. (2006). Problem posing experiences and use of open-ended questions in Mathematics teaching. *Kastamonu Education Journal, 14(1),* 129-146.

Arikan, E. E., & Unal, H. (2013). Examination of primary school year 2. students' mathematics problem posing skills. *Ankara University Faculty of Education Journal, 2(2)*, 305-325.

Aydin Unal, Z., & Ipek, A.S. (2009). The effects of realistic mathematics teaching on primary school year 7 students' achievements in multiplying with whole numbers. *Education and Science, 34*(153), 60-70.

Baki, A. (2015). *Mathematics education from theory to practice.* Ankara: Harf Egitim.

Bingolbali, E., & Ozmantar, M. F. (2014). *Mathematical issues encountered in primary education and solution proposals (4.Edition).* Ankara: Pegem Academy.

Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects-state, trends and issues in mathematics instruction. *Educational Studies İn Mathematics, 22*(1), 37-68.

Buyukozturk, S. (2014). *Data Analysis handbook for social sciences. (2th Edition).* Ankara: Pegem.

Cankoy, O., & Darbaz, S. (2010). The effect of problem posing based problem solving on the success of understanding the problem. *Hacettepe University Faculty of Education Journal, 38*, 11-24.

Cohen, J. (1994). The earth is round (p<.05). *American Psychologist, 49*, 997-1003.

Ellerton, N. F. (1986). Children's made-up mathematics problem-A new perspective on talented mathematicians. *Educational Studies in Mathematics, 17,* 261-271.

English, L. D. (1997). The development of fifth-grade children's problem-posing abilities. *Educational Studies in Mathematics Education, 34,* 183-217.

Ercan, B. (2010). *Evaluation of primary school year seven students' knowledge on the concept of whole numbers* (Master dissertation). Accessed on https://tez.yok.gov.tr

Erktin, E., & Nazlicicek, N. (2002). The Shortened Mathematics Attitude Scale for Primary School Mathematics Teachers. *V. National Science and Mathematics Education Congress.* Accessed on 07.03.2016 from the website http://old.fedu.metu.edu.tr/ufbmek 5/b_kitabi/PDF/Matematik/Poster/t194.pdf.

Ersoy, Y. (2005). Progressive attempts on renovating mathematics education-I: Technology assisted mathematics teaching. *The Turkish Online Journal of Educational Technology, 4(2),* 51-63.

Fidan, S. (2008). *The effects of problem posing activities of primary school year 5. students in Mathematics on problem solving achievements* (Master dissertation). Accessed on https://tez.yok.gov.tr

Ghasempuor, Z., Bakar, M. N., & Jahanshahloo, G. R. (2013). Innovation in teaching and learning through problem posing tasks and metacognitive strategies. *Int. J. Ped. Inn., 1(1),* 57-66.

Isguden, E. (2008). *Difficulties encountered by year 7. and 8. students on the whole numbers subject* (Master dissertation). Accessed on https://tez.yok.gov.tr

Kar, T., Ozdemir, E., Ipek, A. S., & Abayrak, M. (2010). The relation between the problem posing and problem solving skills of prospective elementary mathematics teachers. *Procedia Social and Behavioral Sciences, 2,* 1577-1583.

Karakas Turker, N., & Turanli, N. (2008). Developing an attitude scale concerning mathematics education courses. *Gazi Faculty Of Education Journal, 28*(3), 17-29.

Karasar, N. (2014). *Scientific research method (27th Edition).* Ankara: Nobel.

Karasel, N., Ayda, O., & Tezer, M. (2009, May). Effect of problem posing method in mathematic education with computer assisted pictures on student success and attitudes. Paper presented at *9th International Educational Technology Conference*. Ankara-Turkey.

Katranci, Y. (2014). *The effects of problem posing studies in co-operation based learning settings on mathematical understanding and problem solving achievements* (Doctorate dissertation). Accessed on https://tez.yok.gov.tr

Kontorovich, I., Koichu, B., Leikin, R., & Berman, A. (2012). An exploratory framework for handling the complexity of mathematical problem posing in small groups. The *Journal Of Mathematical Behavior, 31,* 149-161.

Koroglu, H., & Yesildere, S. (2004). The effects of teaching primary school year seven mathematics course whole numbers unit through multiple intelligence based teaching on student success. *Gazi Faculty Of Education Journal, 24(2),* 25-41.

Korukcu, E. (2008). *The effects of teaching the whole numbers subject through visual materials on year 6. students' mathematics success* (Doctorate dissertation). Accessed on https://tez.yok.gov.tr

Lavy, I., & Shiriki, A. (2010).Engaging in problem posing activities in a dynamic geometry setting and the development of prospective teachers' mathematical knowledge. *The Journal of Mathematical Behavior, 29,* 11-24.

Lowrie, T. (1999). Free problem-posing: Year 3/4 students constructing problems for friends to solve. *MERGA, 22,* 328-335.

Lowrie, T. (2002). Designing a framework for problem posing young children generating open-ended tasks. *Contemporary Issues in Early Childhood, 3(3),* 354-364.

MoNE (Ministry of National Education) (2013). *Secondary School Mathematics Course (Year 5, 6, 7 and 8) Curriculum.* Ankara: MEB.

Morse, J. M. (1991). Approaches to qualitative-quantitative methodological triangulation. *Nursing Research, 40*, 120-123.

NCTM (2000). *Principals and standards for school Mathematics.* Reston, Va: National Council of Teachers of Mathematics Pub.

OECD. (2006). International student assessment (PISA). Accessed on 20 December 2015 on the website https://data.oecd.org/pisa/mathematics-performance-pisa.htm.

OECD. (2009). International student assessment (PISA). Accessed on 20 December 2015 on the website https://data.oecd.org/pisa/mathematics-performance-pisa.htm.

OECD. (2012). International student assessment (PISA). Accessed on 20 December 2015 on the website https://data.oecd.org/pisa/mathematics-performance-pisa.htm.

Polya, G. (1957*). How to solve it. A new aspect of mathematical method.* Princeton, NJ: Princeton.

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14*(1), 19–28.

Silver, E. A., & Cai , J .(1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematical Research,27*(5), 521–539.

Sahin, A., Ongoren, H., & Cokadar, H. (2010). The effects of multiple intelligence theory based teaching on student attitudes towards science. *e-Journal of New World Sciences Academy, 5(2),* 432-445.

Tavsancil, E., & Aslan, E. (2001). *Content analysis and examples of implementation.* Istanbul: Epsilon.

Tekin, H. (1996). *Assessment and evaluation in education.* Ankara: Yargı.

Toluk Ucar, Z. (2009). Developing pre-service teachers understanding of fractions through problem posing. *Teaching and Teacher Education, 25,* 166–175.

Turgut, U., & Gurbuz, F. (2011). Effects of teaching with 5E model on students' attitudes and conceptual changes about the subject of heat and temperature. *International Online Journal of Educational Sciences, 3(2),* 679-706.

Turhan, B., & Guven, M. (2014). The effects of teaching mathematics through the problem posing approach on problem solving achievement, problem solving skills and mathematics opinions. *Cukurova University Faculty of Education Journal, 43(2)*, 217-234.

Unlu, M., & Sarpkaya Aktas, G. (2017). Examination of pre-service elementary mathematics teachers' problems posed about algebraic expressions and equations. *Turkish Journal of Computer and Mathematics Education, 8(1),* 161-187.

Ustuner, M. (2006). Reliability and validity study concerning the teaching profession attitudes scale. *Educational Management in Theory and Practice, 45,* 109-127.

Varol, F., & Kubanc, Y. (2012). Common arithmetical difficulties that students face on story problems. *Turkish Studies-International Periodical For The Languages, Literature and History of Turkish or Turkic, 7(1),* 2067-2074.

Yenilmez, K., & Bagdat, O. (2014). Learning difficulties of year seven students on whole numbers problems. *Abstracts of International EJER Congress 1st Eurasian Educational Research Congress, 24 – 26 April.*

Yildirim, A., & Simsek, H. (2013). *Qualitative research methods in social sciences (9. Edition).* Ankara: Seckin.

## Problem Kurma Yaklaşımı ile İşlenen Tam Sayılar Konusunun Öğrencilerin Akademik Başarısı ve Matematik Tutumlarına Etkisi

**Atıf:**

Ozdemir, A. S., & Sahal, M. (2018). The effect of teaching integers through the problem posing approach on students' academic achievement and mathematics attitudes. *Eurasian Journal of Educational Research, 78, 117*-138, DOI: 10.14689/ejer.2018.78.6

### Özet

*Problem Durumu*: Ülkemizde 2006 yılından itibaren yapılandırmacı eğitim yaklaşımı benimsenmiştir. 2013 yılında yenilenen matematik programı ortaokul seviyesinde uygulanmaya başlanmıştır. Bu durum sürekli gelişme içinde olan matematik eğitimini de olumlu etkilemiştir. Bunun yanı sıra başka gelişmeler de yaşanmış ve hiçbir zaman kaydedilen ilerlemeler yeterli görülmemiştir.

Uluslararası düzeyde yapılan sınavlara ülkemiz de katılmaktadır. 2006'da, ülkemizde yapılan PISA sonuçlarına göre, matematik performansında erkekler 427 puan alırken, kızlar 421 puan almıştır (OECD, 2006). 2009'daki sınavda erkekler 451 puan alırken kızlar 441 puan almışlardır (OECD, 2009). 2012 yılındaki sınavda ise matematik performansında erkekler 452 puan alırken, kızlar 444 puan almıştır (OECD, 2012). Kaydedilen sonuçlara göre bütün yıllarda alınan bu puanlar Ekonomik İşbirliği ve Kalkınma Örgütü OECD ortalamasının altında kalmaktadır. Yıllara göre kısmen ilerleme kaydedilse de bu ilerlemelerin yeterli olmadığı açıktır.

Programdaki hedeflere ulaşmada başarı elde etmek için hiç kuşkusuz en büyük görev öğretmenlere düşmektedir. Matematik eğitimcilerine geleceği şekillendirmede büyük görevlerin düştüğünü söyleyebiliriz. Çünkü Ersoy (2005) geçen yüzyılda bilim ve teknolojideki ilerlemelerin ve gelişmelerin temel belirleyicisinin matematik olduğunu belirtmiştir. Bu yüzden öğretmenlerin her an değişim halinde olan yaşama ayak uydurmaları elzemdir. Öğretmenlerin öğretim programının hedefleri doğrultusunda sorgulayan, kendini geliştirmeye açık olan, matematiğin değerini anlayan ve takdir eden, kavramlar arası ilişkilere hâkim olan, öğrenmeyi öğrenen öğrenci yetiştirmeyi amaç edinmeleri gerekir.

Öğrenciyi, problem çözme becerilerini geliştirmek suretiyle gerçek hayata hazırlamak eğitim sistemimizin amaçları arasındadır. Programda problem çözme ortaokul matematik eğitiminin her aşamasında önemli görülmüş, bu becerinin geliştirilmesi gerektiği belirtilmiştir (MoNE, 2013). Çünkü problemler yoluyla öğretim öğrencilerin kavramları içselleştirmesine ve becerilerini geliştirmesine yardımcı olmaktadır (Akay, Soybaş ve Argün, 2006). Uyguladığımız programda hayli önemli olan problem nedir? Problem, açık sorular içeren, kişinin ilgisini çeken ve kişinin bu soruları cevaplamak için yeterli tecrübe ve bilgi birikimine sahip olmadığı durumdur (Blum ve Niss, 1991).

Probleme öğrencinin ilgi duyması, problemin merakını kamçılaması ve onun çözümü için güdülenmiş olması gerekir.

Son yıllarda problem çözmenin yanı sıra problem kurma konusuna eğilim artmıştır. Problem kurma, yeni bir problem üretme veya bir problemin yeniden formüle edilmesi olarak adlandırılan eylemdir (Silver, 1994). Problem kurma Polya'nın ortaya koyduğu 4 adımlı sürecin 5. aşaması olarak düşünülmüştür (Gonzales, 1998). Ayrıca NCTM (2000) çağdaş eğitim yaklaşımında problem kurmayı alternatif bir yaklaşım olarak görmüştür. Problem kurma, problem çözme becerisine ve matematiğe karşı tutumlara katkısı bakımından önem arz etmektedir (Silver, 1994; Turhan ve Güven, 2014). Problem anlaşılmaz, ona karşı uygun bir plan düşünülmez ve çözüme ulaşılamazsa matematiğe karşı olumsuz tutum ortaya çıkmaktadır (Cankoy ve Darbaz, 2010). Problem kurma yaklaşımı bu kısır döngüyü bertaraf etmek için kullanılabilir. Çünkü Akay ve Boz (2009) problem kurmanın matematiği sevdirdiğini, problem çözmeyi geliştirdiğini, kavramsal anlamayı kuvvetlendirdiğini, zihinde kalıcılığı artırdığını, demokratik öğrenme ortamı oluşturduğunu ve tek yönlü öğrenmenin dışına çıkarak matematiksel kavramlar arasındaki ilişkileri keşfetmeyi sağladığını belirtmişlerdir. Akay (2006) problem kurma yaklaşımıyla işlenen dersin öğrencilerin akademik başarılarını ve problem çözme becerilerini olumlu yönde etkilediğini bulmuştur. Ayrıca Lowrie (1999) öğrencilere kendi problemlerini oluşturmak için fırsat vermenin onların matematiğe karşı inançlarını ve tutumlarını gözlemlemeye olanak sağladığını belirtmiştir. Yapılan çalışmalar, problem kurmanın matematiğe karşı olan yaygın korkuları ve endişeleri azaltabileceğini göstermektedir (Silver, 1994; English, 1997). Bu bilgiler ışığında problem kurmanın önemli olduğu söylenebilir.

Tam sayılar öğretim programında geniş bir yer tutmaktadır. İlkokulda öğrenciler doğal sayılar ve rasyonel sayıların pozitif kısmıyla karşılaşmaktadır. Ortaokul döneminde tam sayılara geçiş süreci birçok öğrenci için sorun teşkil etmektedir. Literatüre baktığımızda tam sayılar konusunun fazla araştırma yapılan konulardan biri olduğunu görüyoruz (Körükçü, 2008; İşgüden, 2008; Ercan, 2010, Varol ve Kubanç, 2012; Bingölbali ve Özmantar, 2014; Aydın Ünal ve İpek, 2009; Köroğlu ve Yeşildere, 2004). Buna rağmen matematik eğitiminde en fazla zorluk çekilen konulardan biridir (Yenilmez ve Bağdat, 2014). Ayrıca bu konuda yaşanabilecek aksaklıkların, kavram yanılgılarının ve yanlışlıkların gelecek konularda bazı sıkıntılara yol açması muhtemeldir. Çünkü tam sayılar kendisinden sonra gelen birçok konuya temel teşkil etmektedir. Bu bakımdan tam sayılar ortaokul matematik programında kritik bir konu olarak ele alınmaktadır. Yapılan çalışmaların genellikle kavram yanılgılarının ve öğrenci güçlüklerinin üzerinde yoğunlaştığı görülmektedir. Öğrencilerin zorluk çektiği bir konuda araştırmaların çeşitliliği ve derinliği artırılmalıdır. Bu açıdan çalışmanın araştırmacılara, öğretmenlere ve tam sayılar öğretimine katkı sağlayacağı düşünülmektedir.

*Araştırmanın Amacı*: Problem kurmayla ilgili yapılan çalışmaların kurulan problemlerin niteliği, problem kurmanın başarıya ve tutuma olan etkisi, öğretmenlerin ve öğretmen adaylarının problem kurma becerilerini belirlemeye yönelik olduğu görülmüştür. Problem kurma etkinliklerinin özel konuların öğretiminde kullanılmasına yönelik bir çalışmaya rastlanmamıştır. Bu çalışma için problem kurma yaklaşımının tam sayılar öğretimi üzerindeki etkisi merak konusu olmuştur. Bu araştırmacının amacı 6. sınıf tam sayılar konusunun problem kurma yaklaşımıyla işlenmesi sonucunda öğrencilerin yine tam sayılar akademik başarılarında ve matematik tutumlarında kayda değer bir gelişme olup olmadığını araştırmaktır. Bu yönüyle de çalışmanın araştırmacılara ve alana katkı sağlayacağı düşünülmektedir.

*Araştırmanın Yöntemi*: Araştırmada nicel ve nitel araştırma yöntemlerinin bir arada kullanıldığı karma yöntem benimsenmiştir. Çalışmanın nicel boyutunu ön-test son-test kontrol gruplu deneme modeli oluştururken, nitel boyutunda ise gözlem metodundan ve öğrenci çalışma kağıtlarının içerik analizinden yararlanılmıştır. Gönüllülük esasına göre seçilen 34 deney ve 35 kontrol grubu olmak üzere toplamda 69 katılımcı araştırmanın çalışma grubunu oluşmaktadır.

*Araştırmanın Bulguları*: Çalışmadan elde edilen bulgulara göre deney ve kontrol gruplarının akademik başarıları ve matematik tutumları arasında anlamlı bir fark bulunmadığı; uygulama sonrasında ise deney ve kontrol gruplarının akademik başarıları arasında deney grubu lehine anlamlı fark olduğu, matematik tutumlarında ise aralarında anlamlı bir fark olmadığı bulgularına ulaşılmıştır. Yapılan gözlemler neticesinde problem kurma yaklaşımının sınıf içinde tatlı bir rekabet oluşturduğu ve katılımı nispeten artırdığı görülmüştür. Ayrıca öğrenci çalışma kağıtlarından katılımcıların problem kurma becerilerinin geliştiği, hatalarını fark ettikleri ortaya çıkmıştır.

*Sonuç ve Öneriler*: Araştırmanın sonuçları doğrultusunda problem kurma yaklaşımının tam sayılar konusundaki akademik başarıyı olumlu yönde etkilediği, öğrencilerin matematik tutumları üzerinde kayda değer bir etkisinin olmadığı anlaşılmıştır. Çalışmadan ulaşılan sonuçlara göre problem kurma, yapısına uygun konularda ders içi ve ders dışı etkinlikler için matematik eğitimcilerine önerilebilir.

*Anahtar Kelimeler:* Matematik eğitimi, problem kurma, problem çözme, tam sayılar öğretimi.