Problem-Posing Skills for Mathematical Literacy: The Sample of Teachers and Pre-Service Teachers*

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ABSTRACT

Purpose: The present study aims to examine the mathematics teachers’ and the pre-service teachers’ problem-posing skills for mathematical literacy (ML).

Research Methods: This research was carried out using the case study model. The study group consisted of 13 pre-service mathematics teachers and five middle school mathematics teachers who took ML courses in undergraduate and graduate education. In this context, three free problem-posing activities were asked to pose problems for ML from the participants. The problems posed by the participants were examined through descriptive analysis. The theoretical basis in the PISA study was accepted as the framework in data analysis.

Findings: Analyses showed that the problems posed by the participants were mostly open-ended problems. According to the context of the posed problems, occupational and personal context problems were found to be dominant. The findings showed that the posed problems were more about change-relationships and space-shape concerning mathematical content. Regarding mathematical processes, the average score of employing skills was higher than formulating and interpreting skills scores. The findings suggest that the teachers are more successful than pre-service teachers in problem-posing activities for ML.

Implications for Research and Practice: It was understood that there was a need for theoretical and practical courses that clarify the application of ML in different problem types. Challenges to pose unstructured problems for ML should be removed. The situations and content of mathematics used in ML problems should be made more effective and diversified.

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Introduction

In recent years, the concept of literacy is one of the important topics discussed and researched in the field of education. It is possible to say that there are also changes in the meaning of the concept of literacy from the past to the present day. In the Programme for International Student Assessment [PISA] study conducted by the Organization for Economic Co-operation and Development [OECD], the concept of literacy is expressed as follows: “the ability to use, analyze, make logical conclusions, and communicate effectively when describing, interpreting, and solving problems that students encounter in various situations in key subject areas” (Ministry of National Education [MoNE], 2016, p. 1). There are different approaches to literacy, such as current and traditional approaches (Ozgen & Kutluca, 2013). The traditional literacy approach involves skills, such as reading, writing, and performing simple calculations, which require four operations. However, in current literacy approaches, field-specific competencies and skills are seen as the forefront. In particular, the concept of mathematical literacy (ML) is closely related to many other concepts discussed in mathematics education (Stacey, 2010). In the context of ML, the content of mathematics, process-based skills and the contexts in which it is used are handled.

The PISA 2000 report explained that the term literacy was used to clarify the mathematical knowledge and the ability to deliver functional use rather than merely mastering a school curriculum (OECD, 2000). According to Evans (2017), being literate involves using mathematical knowledge meaningfully in different contexts, allowing us to be better in our daily lives and being functional in a technology-rich learning environment. Addressing literacy in all disciplines of the curriculum from the first years of teaching is the key to lifelong learning and quality of life (Evans, 2017). In mathematics education, it is possible to see the importance and validity of ML in direct or indirect approaches, in mathematics curricula (MoNE, 2017) or in standards explained by institutions for school mathematics (National Council of Teachers of Mathematics [NCTM], 2000). In this respect, De Lange (2003) states that pure mathematics is important but insufficient to do mathematics in real life.

Mathematical problem-solving can be seen as a key feature and focus of ML’s process skills. In this context, it is understood that the measurement of ML in PISA studies takes place using problem-solving (OECD, 2013). In the field of mathematics education, it is seen that there has been an increasing interest in problem-posing in recent years because problem-posing is accepted as the last step of the problem-solving process (Gonzales, 1998). In addition, problem-solving requires high-level cognitive skills (English & Sriraman, 2010; Silver, 1987) and is reported to be closely related to problem-posing skills (Cai, 1998; Kilpatrick, 1987). In this study, ML and problem-posing concepts will be emphasized because problem-posing and problem-solving skills can be seen as complementary elements. Moreover, while ML and problem-solving relationships have been examined in many studies (Memnun, Akkaya & Haciomeroglu, 2012; Sumen & Calisici, 2012), there are limited studies on ML and problem-posing relationships (Ozgen, Ozer & Arslan, 2019; Sahin & Basgul, 2018). Various problem-posing studies have been conducted with pre-service teachers, but there are limited studies on problem-posing in the context of mathematics literacy.
Mathematics teachers have an important role and responsibility in ML, which is a framework for the mathematics learning-teaching process because mathematics teachers should be able to imagine many pedagogical practices to improve students’ ML skills or competencies (Machaba, 2018). Teachers’ awareness of the importance of strategies for ML positively reflected the development of mathematics teaching (McMillen, Del Prado Hill & Friedland, 2010). This study focuses on the teachers’ mathematical literacy in problem-posing context since it is an important factor in improving students’ competencies in ML. Thus, it is necessary to understand teachers’ competencies in ML help to design effective ML development trajectories and to close the gap between in-service and pre-service teachers’ problem-posing skills.

It was reported that Turkish students attending the PISA exams had a low achievement level in ML and tended to decline in the recent PISA (MoNE, 2012; 2016). Furthermore, it was reported that the teacher factor is an important component in these low results in PISA exams for ML. In line with these results, various reform steps have been taken by the MoNE to improve goals about ML. Actions, such as changes in instructional programs and teacher education, developing and renewing books, resources and materials, and simulating the types of questions in the transition exams to high school exams, were carried out. In mathematics education, the concepts of problem-solving and problem-posing are discussed with these changes made for ML. In addition, the ML courses were added as a compulsory course to teacher education programs at universities. In this study, mathematics teachers and pre-service teachers’ problem-posing skills are presented after they have taken an ML course in their teacher education program at undergraduate and graduate levels.

In light of the literature, it is expected that in-service teachers and educated pre-service teachers should design or pose mathematical problems for ML and use them in their classes. This study focuses on mathematics teachers and pre-service teachers who have received ML education (undergraduate and graduate level) to pose problem activities for ML. For the purpose of the present research, the following problems were sought:

1. How are the mathematics teachers’ and pre-service teachers’ skills to pose problems for ML? What is the type of difficulty level, context, content and process skills of the problems posed by the participants?

2. Do mathematics teachers and pre-service teachers’ problem-posing skills for ML differ concerning PISA process skills?

The present study aims to examine the mathematics teachers and pre-service teachers’ problem-posing skills for ML. The problems posed by teachers and pre-service teachers for ML are examined concerning problem type, level of difficulty, context, mathematical content, topics and processes. In addition, the problems posed by the participants for ML are compared concerning defined criteria.
Theoretical Framework

Mathematical Literacy (ML)

ML is not only composed of pure mathematical knowledge, but it has complex structures and components (De Lange, 2001; Matteson, 2006; OECD, 2003). In one of the ML approach, Goos (2007) identified five components (context, mathematical knowledge, disposition, critical orientation and tools) for ML (cited in Bennison, 2015). In Pugalee’s (1999) ML model, ML processes are described as representing, manipulating, mathematical reasoning and, problem-solving. In addition, facilitators for ML in the model are specified as communication, technology and values. In the PISA study, the measurement and evaluation of mathematics are shaped around the concept of ML. In the PISA 2012 study, ML is defined as follows:

Mathematical literacy is an individual’s capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens (OECD 2013, p. 25).

As a consequence of this definition, PISA mathematics units start with a description of a situation that can be encountered in real-life (e.g. a map for a trip, original tables of data, plans for a house, a formula for calculating something practical). A series of questions require students to use this knowledge, calculate the quantities and interpret the results (Stacey, 2010).

There are some approaches to what skills and competencies should be in an individual with ML. Kilpatric (2001) states that mathematical ability includes conceptual understanding, operational fluency, strategic capabilities, adaptive reasoning and productive disposition. NCTM (2000) emphasized the importance of ML. There were five basic skills in learning mathematics: ability to solve mathematical problems, mathematical reasoning ability, mathematical relationship ability, mathematical communication ability and mathematical representation. It is stated here that ML means the complex ability required for students (NCTM, 2000).

Three mathematical processes were defined in the context of assessment and evaluation of ML in PISA 2012 (OECD, 2013, pp. 28-30): (1) mathematically formulating situations, (2) employing mathematical concepts, phenomena, processes and reasoning and (3) interpreting application and evaluation of mathematical output.

Mathematical sciences are interested in describing and analyzing quantities, space-shapes, change-relationships, and at the same time, uncertainty (Diaz, 2017). Steen (1990) suggested inspiration in the developmental power of five deep mathematical thinkers, such as dimensions, quantity, uncertainty, shape and change. Mathematical content knowledge is one of the important sub-dimensions that constitute the framework of measuring and evaluating in ML. In PISA, mathematical content and subject areas refer to subjects or parts that regulate the field of mathematics.
Accordingly, at PISA 2012, there are four subject parts or sections that form the mathematical content of ML (OECD, 2013, pp. 33-35): change-relationships, space-shape, quantity and uncertainty-data.

One of the important sub-dimensions that constitutes the framework for assessment and evaluation of ML in PISA is the context. Mathematical contexts, such as personal, occupational, societal and scientific contexts, are defined in PISA. Thus, the problem context is defined as follows: “Context is the information that is contained and, at the same time, surrounds the statement of a mathematical problem that needs to be mathematized. The context and surrounding information might be necessary or unnecessary for the mathematization of the problem but is independent of the problem’s syntax and stimulus” (Salgado, 2016, p.111). To effectively transfer information from one application to another, students must have experience in solving problems in many different situations and contexts (De Lange, 1987). Steen (2001) categorized the context sub-dimension as personal life, school life, work and leisure and the local community.

In the PISA 2012 mathematics test, three types of items formations were used for the paper-pencil test as follows: (i) open-ended or unstructured items, (ii) structured items and (iii) multiple-choice items. In mathematics and science, test items are grouped into units that start by identifying the real-world situation or problem, which includes text, images, graphics, or information on the charts. Then, a few items are relevant to this situation (Stacey, 2010).

Problem Posing

Mathematical problem-solving plays an important role in many theoretical approaches for ML (Goos, 2007; Pugalee, 1999) and in the framework for assessment and evaluation in the PISA studies. Problems involving the modeling process, particularly the problems that require the use of the model, have a wide range in the PISA studies applied in recent years. Stacey (2015) reported that mathematical modeling was the key to ML, but most people rarely participated in the entire modeling cycle. Brown and Schafer (2006) used a modeling approach in teacher education for ML. Modeling approach used as a tool for ML teaching, formulating, analyzing the model to answer the mathematical questions formulated, interpreting and validating mathematical results concerning context, and finally, consolidation elements. ML focuses on students’ ability to effectively analyze, validate, communicate, formulate, solve and interpret ideas in different forms and situations of problems (Lailiyah, 2017). Teachers should integrate content and/or skills to solve problems (Machaba, 2018). Activities developed by PISA provide problem-solving skills, that is, efforts to provide mathematical understanding and development of the student. It also needs to relate contextual problems to mathematical situations in the problem that the student solves with mathematical knowledge.

Problem-posing and problem-solving are of great importance for mathematics and mathematical thinking (Silver, 1987) because it can be said that problem-posing and problem-solving are considered as closely related and complementary process skills
and even basic problem-posing activities have positive effects on students (Silver & Cai, 1996). In addition, problem-posing activities help students to develop their problem-solving skills (English, 1997). In addition, it is stated that problem-posing activities positively affect students’ problem-solving skills (Lowrie, 2002). The necessity of using problem-posing activities both as a goal and a tool in mathematics education emerges (Kilpatrick, 1987).

Stoyanova and Ellerton (1996) classify problem-posing activities as free, semi-structured and structured problem-posing situations. In this classification, it is important to what extent the problem maker is limited. In this study, free problem-posing will be emphasized because it can be said that free problem-posing is a suitable type of activity for individuals with a lack of knowledge and experience in problem posing. There is no limitation in free problem-posing activities. The problem-maker can pose a problem of his/her own choosing from a real or fictional situation. A problem condition is not given. It may be desirable to establish a problem on any subject. Limitations are as small as possible. “Pose a problem with patterns” or “Pose a problem for 6th-grade students” activities are examples of free problem posing.

**Relationship between ML and Problem Posing**

The use of ML and mathematical knowledge gives learners the ability to better understand the content of different curriculum content in many contexts. As a result, teachers need to have mathematics, pedagogical and curriculum knowledge to have a rich personal understanding of literacy, and to believe that teaching is an integral part of the subject (Bennison, 2015, p. 572).

In related literature, Malasari, Herman and Jupri (2017) stated that high school students should have ML skills in formulating, implementing and interpreting mathematics to solve real-world problems. It has been emphasized that teaching these students with ordinary mathematics problems is not enough and that teachers should create ML problems in the teaching of these students. In this context, it is expected that the awareness of mathematics teachers towards ML is high and problem activities are developed. In the related literature, various studies with teachers and pre-service teachers were found about ML concept. In a study by Lestari, Juniati and Suwarsono (2017), the views of high school teachers towards ML, their habits in mathematics teaching, and their beliefs in integrating ML’s mathematics into learning-teaching were examined. In the study conducted by Pettersen and Nortvedt (2018), teachers’ and pre-service teachers’ competence needs in the activities were determined using an item analysis tool. Concerning competencies (communication, devising strategies, mathematization, representation, symbols and formalism, reasoning and argumentation), each of the teachers and pre-service teachers used a tool about mathematical efficiency and efficiency requirements of activities. Saenz (2009) examined the difficulties of Spanish pre-service teachers in solving PISA questions. In a study conducted by Colwell and Enderson (2016), perceptions of pre-service teachers about literacy in mathematics were examined. Pre-service teachers perceived communication, practice and language as literacy in mathematics.
In recent years, intense interest in the ML concept also appears in a variety of studies in Turkey. In particular, studies with mathematics teachers and pre-service teachers are also observed. The ML self-efficacy beliefs of the pre-service teachers were examined using the ML self-efficacy scale developed by Ozgen and Bindak (2008) in a large part of the studies (Memnun, Akkaya & Haciomeroglu, 2012; Ozgen, 2015; Sumen & Calisici, 2016). ML performance of the pre-service teachers (Gurbuz, 2014; Kabaie & Barak, 2016; Tekin & Tekin, 2004), their views and perceptions towards ML (Ozgen & Kutluca, 2013, Pilten, Divrik, Pilten & Ebret, 2016, Sefik & Dost, 2016) were examined. Also, Gurbuz (2014) determined that pre-service mathematics teachers were successful at selecting and categorizing questions similar to the PISA questions at the end of ML education, but failed to achieve the desired result in question writing.

In addition to intensive and comprehensive ML studies with pre-service teachers, it is seen that there are a limited number of studies with teachers in Turkey. Altun and Akkaya (2014) revealed the reasons for failure of Turkish students in PISA with the views of teachers. Genc and Erbas (2017) examined the views of high school mathematics teachers about the mathematics curriculum emphasizing on ML.

In recent years, there have been many studies with teachers, pre-service teachers and students about problem-posing. In one of these studies, Ulusoy and Kepceoglu (2018) investigated pre-service middle school mathematics teachers’ problems posed in a semi-structured problem-posing context concerning contextual and cognitive structures. The results showed that most of the pre-service teachers used all quantitative data given in the semi-structured problem-posing task. Unlu and Sarpkaya-Aktas (2017) investigated the problems posed by the participants about equations and algebraic expressions in their study with the pre-service mathematics teachers. In their study, pre-service teachers were successful in posing problems for algebraic expressions and equations. Pre-service teachers generally posed verbal problems and frequently used daily language in their problems.

In the relevant literature, there are a limited number of studies dealing with ML and problem-posing together. In the study conducted by Ozgen et al. (2019), mathematics literacy and self-efficacy towards problem-posing were examined. The study was conducted with the participation of the mathematics teachers, it was observed that the mathematics literacy and problem-posing self-efficacy of the participants did not differ significantly according to variables, such as gender, graduation status, faculty graduated, and professional experience. The researchers stated that there was a significant relationship between mathematics teachers’ mathematics literacy and self-efficacy towards problem posing. In another study, Sahin and Basgul (2018) investigated mathematics pre-service teachers’ problem-posing skills appropriateness to the nature of PISA. Their findings showed that many of the problems that pre-service teachers posed are appropriate to the nature of PISA. In addition, it was seen that the pre-service teachers generally posed open-ended problems, but they less prefer to pose multiple-choice and short-answer problems. When pre-service teachers’ problems are examined concerning the content, context and process skills, pre-service teachers’ most posed problems were in the content of quantity, in the personal and occupational context and generative skill-oriented. To
our knowledge, there are no studies examining the problem-posing skills of teachers and pre-service teachers, together with ML.

It is understood that the teachers need training for knowledge and application skills for ML. In some studies (Gurbuz, 2014; Yenilmez & Ata, 2013), the development of problem activities for ML was not thoroughly examined, although ML performance, self-efficacy beliefs or views of mathematics teachers and pre-service teachers were examined within ML course or education. In most of the studies conducted, the activities in past PISA exams which were examined within the scope of the problem activities were applied and investigated. In this study, it is thought that the examination of problem-posing skills for ML after mathematics teachers and pre-service teachers’ ML education is given important information for mathematics education.

One of the most important aims of mathematics education in schools is to improve students’ ML competences. In this context, it is expected that mathematics teachers and pre-service teachers have knowledge, skills and experiences towards ML. The problems that are used in the learning-teaching process of developing students’ ML are important. In this direction, mathematics educators need to pose mathematical problem situations to develop and measure ML.

### Method

#### Research Design

This research was carried out using the case study. The multiple case (holistic) design was adopted in this study. This study contains more than a single case. Thus, this study used a multiple case study design (Yin, 2003, p. 46). In this study, problem-posing situations of teachers and pre-service teachers were accepted as multiple cases. This model is used because the present study aims to investigate the mathematics teachers and pre-service teachers’ problem-posing skills for ML extensively.

It is understood that problem-solving and problem-posing are important factors in developing ML skills and experiences of students in their mathematics courses. In this respect, mathematics teachers should have theoretical knowledge about problem-solving for ML and should be able to reflect on practice. In the relevant literature, various quantitative studies have been conducted on the ML skills, awareness and beliefs of pre-service mathematics teachers and students. In this study, the problem-posing skills of mathematics teachers and pre-service teachers are discussed with a qualitative approach. Examining the mathematics teachers and pre-service teachers’ problem-posing skills together and making comparisons reveal the important and original dimensions of this study.

#### Research Sample

This research was carried out under the ML course, which was an elective course in mathematics education undergraduate and graduate level programs. The study group consisted of 13 pre-service mathematics teachers and five middle school
mathematics teachers who took ML courses in undergraduate and graduate education. Pre-service teachers continued their education in the Faculty of Education. The mathematics teachers were graduated from the Faculty of Education. It is compulsory to take mathematics, mathematics education and general culture courses in teacher education in Turkey. Courses, such as teaching methods, material development, measurement and evaluation, teaching practice, history and philosophy of mathematics, are considered as mathematics education courses and graduates should be successful in these courses. Before this study, pre-service teachers had taken mathematics courses and general educational sciences courses related to the teaching and learning approach. Pre-service teachers did not take a course on mathematics education. In addition, mathematics teachers and pre-service teachers in this study did not take any course for mathematical problem-solving and problem-posing before this study. Since all participants needed to have an ML course history, the purposeful sampling method was applied. In the study, participants participated in the problem-posing process individually after the ML course. Participants did not receive any training or courses for ML before research. Thus, it is assumed that the participants have limited knowledge, skills and experience of ML.

Data Collection

Within the scope of the relevant ML course, theoretical foundations for ML were given and applied studies were carried out. In this context, participants were asked to pose three problems for ML. The data collection tool of this research was the mathematical problems that mathematics teachers and pre-service teachers individually posed for ML. To determine the level of knowledge, skills and experiences of teachers and pre-service teachers for ML, a free problem-solving activity was applied. Because of the lack of knowledge and experience of the participants in this study and their lack of training, it was preferred to apply free problem-posing activities to develop their problem-posing skills. Each participant was asked to pose three problems for ML. Problems posed by teachers and pre-service teachers were not subject to any mathematical concept or subject restriction. Participants were asked to pose mathematical problem situations for ML. Mathematics teachers and pre-service teachers were asked to develop problem-posing activities at the end of the ML course. The application with the teachers and pre-service teachers was made in the classroom environment. In practice, the participants were given sufficient time. It was also said that they could use resources, such as the internet, computer and books in problem posing.

Data Analysis

The problems that participants posed for ML were analyzed by descriptive analysis technique in the scope of qualitative analysis. The data obtained from participants were summarized and interpreted according to the predefined theme and, findings were arranged by a direct citation of the problems (Yıldırım & Simsek, 2005). The themes used as the theoretical framework in the analysis of the problems were determined by examining the PISA 2012 study (OECD, 2012).
In the data analysis, the theoretical basis in the PISA study was accepted as the framework. The mathematical problems that mathematics teachers and pre-service teachers posed in the research were classified as open-ended, multiple-choice, true-false and mixed. Moreover, when the mathematical problems posed by the participants were evaluated according to their difficulty levels, the criteria used in the PISA study were adopted. The mathematical problems posed in this context are considered at six levels concerning difficulty level: Level 1 (L1), Level 2 (L2), Level 3 (L3), Level 4 (L4), Level 5 (L5) and Level 6 (L6).

Mathematical problems were classified as personal, professional, social, and scientific when examined concerning context. In addition, the mathematical problems posed by the mathematics teachers and the pre-service teachers were considered under four components about mathematical content as follows: change-relationships, space-shape, quantity, uncertainty and data.

PISA 2012 provides four mathematical content areas for ML measurement and evaluation, and subheadings for them are also provided (OECD, 2003, p. 36): Functions, algebraic expressions, equations and inequalities, coordinate systems, relations between two and three dimensional geometric objects, measurement, numbers and number sets, arithmetic operations, percentage, rate and fractions, counting principles, estimation, data collection, presentation and interpretation, data diversity and definition of this diversity, sample and sampling, chance and possibility issues constitute the sub-title headings. The problems posed in this study were described concerning sub-topics within mathematical content.

Figure 1. PM977Q02 DVD Rental Question 2 with Categorization (OECD, 2013)
Problems were investigated under three components concerning using mathematical processes: (i) formulating situations mathematically, (ii) employing mathematical concepts, facts, procedures, and reasoning employing mathematical concepts, facts, processes and reasoning, and (iii) interpreting, applying and evaluating mathematical outcomes.

There is an example of one item in the PISA framework in Figure 1. PM977 DVD rental was a three item-unit, which was used in the PISA 2012 field trial then released (OECD 2013). Figure 1 shows the stimulus, Question 2, and the categorization of this question. The item was of above-average difficulty.

In the analysis of the posed problems, descriptive statistics were used. It is possible to have more than one mathematical process skill in a mathematical problem. In other words, one or more of the skills of formulating, employing and interpreting can be found at different levels. Thus, the following points were applied to the problems when the posed problems were examined concerning process skills (formulating, employing and interpreting). A scoring framework to measure participants’ formulating, employing and interpreting process skills are reflected in the problems posed by the participants is presented below. This scoring approach was adapted from PISA studies. As in the PISA study, it was accepted that each problem posed had a dominant process skill. In addition, this dominant process skill was scored according to the degree of reflection of that skill to the posed problem, as in the following rubric.

2 point: Related process skill at the appropriate level
1 point: Related process skill at partially seen
0 point: Empty or no related process skill at all

Participants’ process skills (formulating, employing and interpreting) were examined and compared with the Mann-Whitney U test. Evidence is provided for the themes and categories indicated by the approach of direct citation from the problems of teachers and pre-service teachers. Codes, such as “T1, T2 ...” were given to mathematics teachers and “S1, S2 ...” to pre-service teachers. The data obtained from the posed problems were investigated by the researchers at different times and the reliability of the analysis of the data tried to be provided. Particularly to further refine the categories in the descriptive analysis, participants were directly quoted from the problems they had designed, and comments were presented.

Procedure

This research was conducted in ML courses that participants enrolled as a part of their undergraduate and graduate programs. Teachers and pre-service teachers did not have any interaction with each other. Course processes were conducted at different times and environments. The course practice was conducted by the researcher in a classroom setting and during a semester of three hours a week.

During ML courses, theoretical information was given firstly and then they were asked to make various applications both inside and outside the classroom. Sample
problems for ML were shown by the researcher, and various applications were made by the participants. The sample problems that had emerged in PISA studies in the past years were examined. The expectation at the end of this course was that the knowledge, skills and experience of ML of teachers and pre-service teachers were developed. Participants were expected to develop effective problem-posing skills for the ML. Table 1 provides explanations for what was done in the ML course.

Table 1

<table>
<thead>
<tr>
<th>Contents of ML Course</th>
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<tbody>
<tr>
<td><strong>Weeks</strong></td>
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<td>14</td>
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</table>

Results

In this section, the data obtained in this study were analyzed and presented with tables and figures. The participants posed problems were examined concerning the types of problems and findings are shown in Table 2.

Table 2

<table>
<thead>
<tr>
<th>Distribution of Types of Problems Posed by Participants</th>
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<tr>
<td><strong>Group</strong></td>
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<tr>
<td></td>
</tr>
<tr>
<td>Pre-service Teacher</td>
</tr>
<tr>
<td>Teacher</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

When the problems that the mathematics teachers and pre-service teachers who participated in this research were examined concerning problem types, they were
mostly open-ended problems and the least posed problem type was true-false problems were posed. This finding was found to be valid for both mathematics teachers and pre-service teachers. In the case of mixed problems, two or three problems presented depending on a given situation can cover open-ended, multiple-choice, or true-false types. The multiple-choice problem named “Restaurant”, which the T1-coded teacher posed, is given below. This problem was considered to be mixed in structure because the participant developed both multiple-choice and open-ended problem examples depending on the situation given here.

Restaurant

A fish restaurant manager is looking for a place suitable for this job. He decides to rent one of the places he looks at. The empty area that he finds is 260 m². The manager, who wants to fill the empty area as much as possible, has added as many tables as possible to the area. The dining tables are for four people and have dimensions of 90 cm and 120 cm.

Problem-1: Which of the following can be the number of customers on a day when the restaurant is full?
A) 80  B) 480  C) 2800  D) 4000

Problem-2: The restaurant has 12 kinds of fish, three kinds of salads and five kinds of appetizers. How many different options do customers have if they choose one portion from fish, salads and appetizers?

The example of the open-ended problem named “Discount”, which the S10 pre-service teacher posed for ML, is shown below.

Discount

Suit dress price range: 50 – 500 TL
Jacket price range: 100 - 300 TL
Trousers price range: 30 – 100 TL

A clothing store plans to give a special discount on Christmas. The store team has planned different discounts on suit dress dresses, jacket and trousers. 30% discount on the suit dress, 50% discount on the jacket, and “Buy 3 pay for 2” campaigns on the trousers. Which type of dress has the highest discount rate?

In this study, the participants preferred to pose open-ended problems concerning the type of problems they posed for ML. This finding can be interpreted as a result of the positive effects of the course participants’ course on ML. Because of the ML course, it can be said that they have developed a positive perception about the nature, structure, development and measurement of the problem type suitable for ML.

Findings for the analysis of the participants' difficulty level of the problems that they posed for the ML are presented in Table 3.
Table 3

Distribution of Posed Problems concerning Difficulty Level

<table>
<thead>
<tr>
<th>Group</th>
<th>Difficulty Level</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L6</td>
<td>L5</td>
</tr>
<tr>
<td>Pre-service Teacher</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Teacher</td>
<td>3</td>
<td>21.4</td>
</tr>
<tr>
<td>Total</td>
<td>3</td>
<td>5.8</td>
</tr>
</tbody>
</table>

Participants were found to have problems at most in L4 and L3, and at least L6 concerning the level of difficulty. In addition, there was no problem at L1 concerning the level of difficulty. The difficulty levels of the problems that mathematics teachers posed seem to be higher than the problems of the pre-service teachers. The problem named “Flora” evaluated at the 5th level developed by the S10 pre-service teacher is presented in Figure 2.

![Flora's Problem](image)

**Figure 2. Direct Citation from the Problem of S10**

*Flora*

The above figure symbolizes a mountain. The mountain area is divided into four areas. Various plant species are grown in these areas. As the oxygen rate is decreasing as you ascend up the mountain, the vegetation density gradually decreases, but the number of plant species is equal in all parts of the mountain.

Part 1 of the mountain ..... A1+A2+A3+A4=x

Part 2 of the mountain ..... B1+B2+B3+B4=x

Part 6 of the mountain ..... F1+F2+F3+F4=x
In addition, region 2 has more than 10 plant species in zone 1. Zone 3 has more than 30 plant species in zone 2 and zone 4 has more than 60 plants in zone 3. Accordingly, what is the minimum number of x?

S10’s problem is related to the content of change-relationships and mathematical issues, such as algebraic expressions. In this problem, it can be said that the student must construct a model, think different mathematical concepts together and use different notations. In addition to making algebraic operations with the model that the learner will take the interpretation of the results, he finds as important points. This problem also aims to interpret the findings of the learners. If all these cognitive processes mentioned in this problem are taken into consideration, it can be said that the problem mentioned is a problem that is in accordance with L5 in terms of difficulty level because students at the 5th level can develop and employ models for complex situations. Students can also choose, compare and evaluate appropriate problem-solving strategies. They can communicate between their own interpretations and their conclusions based on their reasoning (OECD, 2013, p. 41).

The problem named “Sales”, in which the S2 pre-service teacher posed was assessed in L2, is shown in Figure 3.

**Problem:** How many more physics books need to be sold so that the number of physics books sold equals the number of mathematics books sold in a week?

S2’s Sales problem includes a graphical representation. In this context, the graph shows the percentage of sales of textbooks in one week. In the problem, it is stated that 1000 books are sold in one week, and according to this, the number of books in all courses can be found by reading the graph. Later, a relationship between the number of physics and mathematics books was questioned. In this case, students have the opportunity to reason about simple relationships in a given situation and to interpret the results in a limited way. Since this problem is not reflected in the upper-level cognitive process skills. It is thought that the problem is an L2 problem concerning the level of difficulty because the students in the second level can apply basic algorithms,
formulas, operations and familiar rules. It has reasoning capacity directly related to the simple relationships seen at first sight and can interpret the results in a limited way (OECD, 2013, p. 41).

To conclude, the participants posed problems with a moderate difficulty level for ML. In particular, participants developed fewer problems in L5 and L6, which suggests that participants have limitations and difficulties in developing high-level problems for ML. The findings of the participants in the survey concerning contexts of the problems they developed for ML are presented in Table 4.

**Table 4**

*Distribution the Posed Problems concerning Context*

<table>
<thead>
<tr>
<th>Group</th>
<th>Personal</th>
<th>Occupational</th>
<th>Societal</th>
<th>Scientific</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>f</td>
<td>%</td>
<td>f</td>
<td>%</td>
<td>f</td>
</tr>
<tr>
<td>Pre-service teacher</td>
<td>8</td>
<td>21.6</td>
<td>19</td>
<td>51.3</td>
<td>2</td>
</tr>
<tr>
<td>Teacher</td>
<td>2</td>
<td>14.2</td>
<td>11</td>
<td>78.5</td>
<td>-</td>
</tr>
<tr>
<td>Total</td>
<td>10</td>
<td>19.6</td>
<td>30</td>
<td>58.8</td>
<td>2</td>
</tr>
</tbody>
</table>

The problems posed by the participants had the most occupational context and at least the social context regarding the context. Pre-service teachers posed more problems in scientific context than teachers. On the other hand, teachers posed more problems in occupational context than pre-service teachers. Besides, the number of problems in societal context is fewer than another context. The problems posed by the participants differed according to the type of context. In social and scientific contexts, participants developed fewer problems than personal and occupational contexts. These findings suggest that participants have some difficulties and limitations in posing contextual problems. It can be said that pre-service teachers prefer to pose more problems in scientific context due to their courses, training and current knowledge. On the other hand, the resistance of teachers to posing different problems may be influenced by the routines they use for a long time in their instructions. The problem of the S6 pre-service teacher posed in the personal context is given below.

**Problem**

Entrance fees for a visit to a zoo with 170 kinds of animals are 5 TL for the students and 10 TL for the adults. This zoo has earned 3,000 TL a day and the number of students who come to the zoo is more than the number of adults. According to this, at least how many students came to the zoo?

The problem that the S6-coded pre-service teacher posed was handled as a personal context concerning the situations in which mathematics was used because, in the personal context, there are categories about the individual himself, his peers and his family. In this context, there are situations related to food preparation, shopping,
play, personal health, travel, personal budget and time management (MoNE, 2012). In this problem, there is an action or behavior in the routine life of the person concerned with the situations related to entrance fees to the zoo. For these reasons, the posed problem is considered as a personal context. The occupational context problem of the T3-coded teacher is given below.

Field

The farmer Mehmet has a field of 200 m and 100 m edge length in the form of a rectangle. Mehmet wants to plant wheat in this field. The amount of seeds proposed by the agricultural engineers for the 1-acre field is a minimum of 20 kg and a maximum of 25 kg.

Problem -1: How many kilograms of seed will be used for Mehmet's wheat cultivation?

Problem-2: Mehmet plows the field with a 4 m long cultivator parallel to the long edge of the field. The second version runs parallel to the short edge.

Select “True” or “False” for the following proposals.

- The cultivator has made fewer round trips in the 1st round of farming
- In the 2nd version of the farm, the cultivator spent less fuel.
- The number of round trips in the 2nd version of the cultivator farm is half of the 1st crop.

There are a real dutiful occupation and related situations in T3’s “Field” problem. This problem is particularly related to the professional situation of farmers and the use of mathematics. Because of this context, there are often situations that are related to a profession or business life. Topics, such as measurement, cost, ordering for buildings, accounting, quality control, time management, design/architecture, business-based decisions, are evaluated within the professional context (MoNE, 2012). In this problem, it can be said that the situations in the farming profession and the topics, such as length, area and measurement from traditional mathematics subjects, are related. It is considered that this problem developed due to these reasons is in the occupational context.

Some of the contexts were less involved in the problems posed by participants in this study for ML. This finding may be an indication of the participants' limited knowledge and beliefs about the use of mathematics. The participants in the research were more successful in writing problems in professional contexts. Pre-service teachers were able to establish problems in different contexts, while teachers were not able to establish problems in all contexts. Different experiences are needed to develop contextual problems for ML.

Mathematics teachers and pre-service teachers’ problems that they posed for ML are analyzed in terms of mathematical content and the findings obtained are shown in Table 5.
Table 5

Distribution of Posed Problems concerning Mathematical Content

<table>
<thead>
<tr>
<th>Group</th>
<th>Change-relationships</th>
<th>Space-shape</th>
<th>Quantity</th>
<th>Uncertainty-data</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>f</td>
<td>%</td>
<td>f</td>
<td>%</td>
<td>f</td>
</tr>
<tr>
<td>Pre-service teacher</td>
<td>21</td>
<td>56.7</td>
<td>6</td>
<td>16.2</td>
<td>3</td>
</tr>
<tr>
<td>Teacher</td>
<td>4</td>
<td>28.5</td>
<td>5</td>
<td>35.7</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td>25</td>
<td>49</td>
<td>11</td>
<td>21.5</td>
<td>6</td>
</tr>
</tbody>
</table>

Analysis of the participants’ problems in terms of mathematical content, it was revealed that the content of the change-relationships was most frequently used, while the content of the quantity was the least frequently used. It can be said that teachers’ and pre-service teachers’ problems did not show similarity in terms of the problem content. In particular, it is seen that teachers place the most space-shape in their problems and at least the uncertainty-data content. The problem named “Desktop Computer Set”, which S12-code pre-service teacher posed in the context of change-relationships, is given below.

**Desktop Computer Set**

Furkan wants to buy a desktop computer set, and, going to a store selling computers and parts. This store has a sales campaign. In this campaign, a monitor a case, a keyboard, a mouse, and two speakers can all be taken together with a more appropriate price tag. The prices of this store are given in Table 6.

Table 6

The Prices of Stores

<table>
<thead>
<tr>
<th>Product</th>
<th>Price (TL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Campaign products</td>
<td>1700 or 2100</td>
</tr>
<tr>
<td>Monitor</td>
<td>500, 645 or 900</td>
</tr>
<tr>
<td>Case</td>
<td>750</td>
</tr>
<tr>
<td>Keyboard and mouse</td>
<td>320 or 450</td>
</tr>
<tr>
<td>1 speaker</td>
<td>65 or 110</td>
</tr>
</tbody>
</table>

**Problem-1**: Furkan wants to buy these products separately or use the campaign. What is the lowest and highest price for this?

**Problem-2**: The store offers three different monitors, two different keyboards and a mouse, two different speakers. There is only one option for the case. How many different desktop computers can Furkan create?

**Problem-3**: Since Furkan can spend 1975 TL, he wants to buy the most expensive desktop computer set he can get with his money. So how can this selection be picked up?
In the problem named “Desktop computer set” posed by the pre-service teacher with S12 code, there are various computer parts and alternative prices for them. Here, if the student gets a desktop computer, the minimum or maximum wage account is asked. In addition, how many different combinations can be made is asked. This problem is directed at examining the changes and relationships within a situation that students may encounter in the real world because this problem emphasizes the changes and relationships between both the real world and the edited event, phenomenon, or situations. The contents of change-relationships include functions of algebraic expressions, equations, inequalities, table and graphical representations and algebraic topics from traditional mathematical subjects (OECD, 2013, pp. 33-35). In this direction, it can be said that the students are able to recognize the change-relationships in the given situation and transform them into a mathematical world, where the mathematical process skills are to be employed on and the interpretation of the findings obtained later is possible. This problem can be considered in the change and relationships in terms of mathematical content. The problem titled “Article” which S7 code pre-service teacher posed in space-shape content is given below.

Article

The teacher wants students to write an article on rectangular paper with a short edge of 16 cm and a long edge of 35 cm. The teacher wants a space of 2 cm from the top, a space of 3 cm below, and a space of 1.5 cm from the right and left edges to be left blank. Considering these precautions, how many square centimeters are the areas of the smallest rectangular area to enclose the article to be written?

It is required to write an article under certain conditions on a paper with a rectangular shape given the length of the problem named “Article” posed by the S7 code pre-service teacher. In addition, the area of the smallest rectangle that can contain this article is questioned. In this problem, the smooth geometric shape deals with the situation of reflection in the real world. Since the problem is related to geometry, it can be said that this problem is related to the space-shape content in terms of mathematical content. This is because space-shape content deals with phenomena that appear frequently and are frequently encountered in the physical world. Phenomena, such as patterns, properties, locations and centers, representations, coding and recoding of visible information, directions of real shapes and dynamic interactions, are in the space-shape content. The space-shape content generally falls into the field of geometry. Space-shape content includes actions, such as perspective drawings, map drawings, drawing and transforming shapes, three-dimensional views, representation of shapes (OECD, 2013, pp. 33-35).

The problems posed by the participants in this study did not have a balanced distribution concerning mathematical content. Particularly, the participants did not mostly prefer to develop problems in terms of quantity and uncertainty-data content. This finding may be due to the difficulty of the nature of the mathematical concepts in these contents or the beliefs of the participants about these contents. It was understood that the participants in the research found and posed real-life problems more easily in
the fields of change-relationship and geometry. Participants may have difficulties in finding and establishing real-life problems in other content areas. Another reason might be the nature of PISA contents. Because participants have a course on PISA and ML, they may tend to pose a problem on change and relationship. PISA includes more problems with change and relationship than uncertainty.

The findings of the mathematics teachers and pre-service teachers in the study are examined in terms of the traditional mathematical subject in ML. The findings are shown in Table 7.

Table 7

Distribution of Posed Problems concerning Mathematical Subject

<table>
<thead>
<tr>
<th>No</th>
<th>Mathematical subject</th>
<th>Pre-service teacher</th>
<th>Teacher</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>f</td>
<td>%</td>
<td>f</td>
</tr>
<tr>
<td>1.</td>
<td>Functions</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>2.</td>
<td>Algebraic expressions</td>
<td>1</td>
<td>2.7</td>
<td>-</td>
</tr>
<tr>
<td>3.</td>
<td>Equations and inequalities</td>
<td>8</td>
<td>21.6</td>
<td>2</td>
</tr>
<tr>
<td>4.</td>
<td>Coordinate system</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5.</td>
<td>Relations between 2D and 3D geometric objects</td>
<td>5</td>
<td>13.5</td>
<td>5</td>
</tr>
<tr>
<td>6.</td>
<td>Measuring</td>
<td>1</td>
<td>2.7</td>
<td>1</td>
</tr>
<tr>
<td>7.</td>
<td>Numbers and number sets</td>
<td>2</td>
<td>5.4</td>
<td>1</td>
</tr>
<tr>
<td>8.</td>
<td>Arithmetic operations</td>
<td>6</td>
<td>16.2</td>
<td>1</td>
</tr>
<tr>
<td>9.</td>
<td>Percent, rate and fractions</td>
<td>5</td>
<td>13.5</td>
<td>1</td>
</tr>
<tr>
<td>10.</td>
<td>Counting principles</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>11.</td>
<td>Estimation</td>
<td>1</td>
<td>2.7</td>
<td>-</td>
</tr>
<tr>
<td>12.</td>
<td>Data, collection, presentation and interpretation</td>
<td>3</td>
<td>8.1</td>
<td>-</td>
</tr>
<tr>
<td>13.</td>
<td>Data diversity and identification of this diversity</td>
<td>3</td>
<td>8.1</td>
<td>2</td>
</tr>
<tr>
<td>14.</td>
<td>Sample and sampling</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>15.</td>
<td>Chance and probability</td>
<td>2</td>
<td>5.4</td>
<td>-</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>37</td>
<td>100</td>
<td>14</td>
</tr>
</tbody>
</table>

The problems posed by the participants included the most equations and inequalities and the relations between 2 and 3-dimensional geometric objects. Besides, sample and sampling, counting principles, coordinate systems, etc. have not been found to have any problems. Functions, algebraic expressions, estimation, etc. are seen to be the subjects that are included in the least number of problems.

In this study, the problems developed by participants in ML were different in terms of mathematical subjects. However, in parallel with the previous findings, the subjects of change-relationships and space-shape contents were more involved in the problems. It was found that the subjects included in the quantity and uncertainty-data contents were less involved in the problems.
Mathematics teachers and pre-service teachers’ problems that they posed for ML are examined in terms of mathematical processes, and the findings obtained are presented in Table 8.

Table 8

<table>
<thead>
<tr>
<th>Group</th>
<th>Formulating</th>
<th>Employing</th>
<th>Interpreting</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
<td>M</td>
<td>SD</td>
</tr>
<tr>
<td>Pre-service teacher</td>
<td>2.15</td>
<td>1.21</td>
<td>4.38</td>
<td>1.60</td>
</tr>
<tr>
<td>Teacher</td>
<td>4.40</td>
<td>0.89</td>
<td>5.60</td>
<td>0.89</td>
</tr>
<tr>
<td>Total</td>
<td>2.77</td>
<td>1.51</td>
<td>4.72</td>
<td>1.52</td>
</tr>
</tbody>
</table>

The participants in this study used the most employing and least formulating mathematical processes in the problems they posed for ML. Teachers and pre-service teachers were found to have a higher average of employing than other process skills.

The problem of the S1’s named “Competition”, which includes the skills of employing and interpreting from mathematical process skills, is shown in Table 9.

Table 9

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Material Usage</th>
<th>Time</th>
<th>Cooking form</th>
<th>Nutritive value</th>
<th>Taste</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scores</td>
<td>20</td>
<td>20</td>
<td>15</td>
<td>30</td>
<td>25</td>
</tr>
</tbody>
</table>

In the food competition, which is given by the above table cooks, there are the highest scores for the five criteria. Ranking in the competition will be based on the sum of the points the competitors get from each criterion. In case of an equality of the total scores, the competition with a high score from nutritional value criterion will come forward. The scores of the five chefs at the end of the competition are given in Table 10.

Table 10

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Material Usage</th>
<th>Time</th>
<th>Cooking form</th>
<th>Nutritive value</th>
<th>Taste</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gizem</td>
<td>5</td>
<td>20</td>
<td>10</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>Demir</td>
<td>10</td>
<td>20</td>
<td>10</td>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>Fatma</td>
<td>5</td>
<td>20</td>
<td>10</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Semih</td>
<td>10</td>
<td>20</td>
<td>15</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>Duru</td>
<td>5</td>
<td>20</td>
<td>10</td>
<td>30</td>
<td>10</td>
</tr>
</tbody>
</table>

Problem-1: If the competitor with the lowest total score had received a full score from the nutritional value criterion, what would be the ranking?
Problem-2: When we rank scores from highest to lowest, what happens to Semih in the competition?

Problem-3: Indicate “True” or “False” for each of the following statements.

a.) If Gizem received full marks from the criteria for material use, it would have passed the Damla.

b.) If Fatma took 5 points more than the nutritional value criterion, it would be in the same order as Semih.

c.) It would be in the same order as Semih if the score Gizem received from the taste criteria, and the score it received from the nutritional value were replaced.

It can be said that in the problem named “Competition” posed by the pre-service teacher with S1 code, the skills of employing and interpreting take place in terms of mathematical processes. In this problem, mathematical operations, such as arithmetic summing and table reading, are required, and these operations describe the mathematical ability to employ. Because employing describes the use of mathematical concepts, facts, processes, and reasoning of individuals to solve mathematically formulated problems to obtain a set of mathematical decisions (OECD, 2013, pp. 29-30). Moreover, it can be said that the results or decisions obtained as a result of the employing process reflect the interpreting skills in the context of the real-life i.e., the problem. Interpreting is defined as the capacity of individuals to interpret them in a real-life problem by showing mathematical solutions, conclusions, or decisions. This mathematical process can also be defined as an evaluation process (OECD, 2013, pp. 29-30). The “Gold Mine” problem, which is posed by the T2 code mathematics teacher and includes the skills of formulating, employing and interpreting, is given below.

Gold Mine

A company that tries to find a gold mine collects samples by making excavations. The depths in which the four collected samples are taken are given in Table 11.

Table 11

<table>
<thead>
<tr>
<th>Sample</th>
<th>Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>120 m</td>
</tr>
<tr>
<td>L</td>
<td>50 m</td>
</tr>
<tr>
<td>M</td>
<td>98 m</td>
</tr>
<tr>
<td>N</td>
<td>80 m</td>
</tr>
</tbody>
</table>

Problem-1: How many meters do the average of the depths of gold samples taken according to the table?

Problem-2: The miners could go down to 90m deep in the first week of their work. According to which gold samples they reached?

A) K-L  B) L-M  C) M-N  D) L-N

Problem-3: How many meters in depth could a miner who reached the M gold sample but could not reach the K sample?

A) 121  B) 119  C) 97  D) 90
It can be said that in the problem named “Gold Mine” posed by the teacher with T2 code, the skills of formulating, employing and interpreting take place in terms of mathematical processes. Students need to establish and formulate a mathematical structure to solve this problem. This is because formulating means that individuals can recognize situations in which they can use mathematical knowledge and skills and, after their definition, can make mathematical constructs for a problem presented or presented in the theoretical context. Formulation demonstrates the ability of individuals to analyze, construct and solve a problem and to reveal basic mathematical knowledge and skills (OECD, 2003, p. 28). Later, mathematical manipulation within the mathematical structure established by the students is considered as the ability to employ. Because mathematical concepts, facts, processes, and reasoning in problem-solving carry out many mathematical operations that require individuals to get results and find solutions in the process of employing. For example, arithmetic addition, equation solving, reduction based on mathematical assumptions, table and graph reading, data analysis can be seen as such mathematical operations (OECD, 2013, p. 29). The ability of the students to evaluate the results and decisions they have made for the given problem reflects the interpretation process. Assessment requires that certain results or solutions be produced. In the context of PISA ML, the interpretation process refers to the interpretation of these results or solutions as they are transferred to a real-life situation (OECD, 2013, pp. 29-30).

In this study, employing skills are given more concerning mathematical process skills of the problems developed by participants in ML. Formulation and interpretation skills are also very important for ML, but they have been less involved in developed problems. This finding may be interpreted as a result of the operational oriented viewpoints and beliefs of the participants in mathematics and mathematics education. Findings for comparison of mathematical process skill mean scores for the problems that teachers and pre-service teachers have developed are shown in Table 12.

**Table 12**

*Mann Whitney-U Test Results of Mathematical Process Scores of Participants*

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>Mean Rank</th>
<th>Sum of Ranks</th>
<th>U</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formulating</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-service teacher</td>
<td>13</td>
<td>7.15</td>
<td>93.0</td>
<td>2.00</td>
<td>.002*</td>
</tr>
<tr>
<td>Teacher</td>
<td>5</td>
<td>15.60</td>
<td>78.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employing</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-service teacher</td>
<td>13</td>
<td>8.35</td>
<td>108.50</td>
<td>17.50</td>
<td>.113</td>
</tr>
<tr>
<td>Teacher</td>
<td>5</td>
<td>12.50</td>
<td>62.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interpreting</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-service teacher</td>
<td>13</td>
<td>8.19</td>
<td>106.50</td>
<td>15.50</td>
<td>.086</td>
</tr>
<tr>
<td>Teacher</td>
<td>5</td>
<td>12.90</td>
<td>64.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-service teacher</td>
<td>13</td>
<td>7.46</td>
<td>97.0</td>
<td>6.00</td>
<td>.009*</td>
</tr>
<tr>
<td>Teacher</td>
<td>5</td>
<td>14.80</td>
<td>74.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p<.05*
It was found that there was a statistically significant difference in the total scores of the mathematical process skills ($U = 6.00; p < .05$) and the formulating scores ($U = 2.00; p < .05$) for the teachers and pre-service teachers. It seems that this difference in formulating and total points is in favor of the teachers. According to this finding, it can be said that teachers reflect mathematical process skills better in problems. Moreover, there was no significant difference between employing ($U = 17.50; p > .05$) and interpreting scores ($U = 15.50; p > .05$) of teachers and pre-service teachers from mathematical process skills. Thus, it can be said that teacher and pre-service teachers reflect the skills of employing and interpreting from mathematical process skills at similar levels in problems. Teachers were ahead of formulating skills and overall total process skills scores compared to pre-service teachers. An underlying reason for this finding may be explained that teachers are more experienced in formulating skills.

**Discussion, Conclusion and Recommendations**

In this study, the problem-posing skills of mathematics teachers and pre-service teachers who took the ML course were examined. The problems that participants posed for the ML were examined within the scope of the ML measurement and evaluation criteria of the PISA.

In the analysis of the obtained data, it was seen that the problems posed in general were open-ended problems according to problem type. The vast majority of the problems that mathematics teachers and pre-service teachers develop were open-ended problems. After open-ended problems, multiple-choice problems were developed more in number. There were very few problems in the true-false type. This is demonstrated by the participants that open-ended problems are important in the measurement and development of ML. This is because the process, content and context skills that are targeted within the ML are dependent on the nature of the problems and problems being addressed. Sahin and Basgul (2018) found that mathematics pre-service teachers usually posed open-ended problems, but they less preferred to pose multiple-choice and short answer problems after the ML training. Moreover, this result of this study can be seen as the positive reflections of the course that the participants take for ML because of the nature and assessment of ML are compatible with open-ended problems. It is also seen in the findings of this study that it would not be appropriate to use predominantly multiple-choice or true-false problems to develop or measure ML skills. In another study in related literature, factor analysis was carried out concerning the structural features of the ML questions in the study conducted by Altun and Bozkurt (2017). The six-factor structure of the question has been put forward in the form of algorithmic processing, the dominance of rich mathematical content, mathematical inference, mathematical proposal development and/or improved proposal interpretation, comprehension of vital state in mathematical language, the understanding mathematical language in life. It can be argued that the goals of the open-ended structure of the problems that are developed in the teaching, or measurement within the ML are one of the facilitating indicators.
In this study, the problems developed by the participants were mostly concentrated at L4 about the level of difficulty. It was understood that the problems posed by the participants were at the level of L4 and later on the L3. At the highest levels, L6 and L5, a limited number of problems posed by both teachers and pre-service teachers. However, it can be said that mathematics teachers are one step ahead of developing difficult problems concerning difficulty level. More structured problems were posed for ML by the participants. On the contrary, unstructured problems are also known to be needed. In other studies carried out in Turkey, the situation is seen similar to the results observed in this study. Ulusoy and Kepceoglu (2018) found that only 8% of the problems posed by pre-service middle school mathematics teachers at reasoning level. Pre-service teachers produce procedural questions at the application level that could be solved mostly by multi-step processes rather than by reasoning. The results of this study revealed that the pre-service teachers thought that as the operational load of problems increases, the difficulty and cognitive levels of problems also increase. In another study, Iskenderoglu and Baki (2011) classified the questions in one of the 8th-grade textbooks according to the PISA mathematics proficiency scale. According to the results of this classification, it was found that there were no questions at all levels in the 8th-grade textbook. In the book examined, questions, problems, exercises and examples were found to be at Levels 1, 2, 3 and 4. Questions are mostly at Level 2 in this study. Also, Iskenderoglu, Erkan and Serbest (2013) categorized and examined the mathematics questions used in transition to high school exams implemented in Turkey according to the PISA proficiency scale. The mathematics questions in the examinations in the study did not seem to be appropriate for all levels. The questions were generally taken on levels 2, 3 and 4. There was 1 question at the 5th level, which was the highest level and no problem in the 6th level. The results obtained in this study and the corresponding levels on similar results in the literature are similar to ML results of the PISA study in Turkey. The average of the ML scores of the Turkish students in the PISA 2012-2015 exams was in the range of 2 and 3 level (MoNE, 2012, 2016). When these results are considered together, the mathematics education in Turkey has manifestly failed to reach the top levels 5 and 6 in the PISA exams. In addition, it can be said that the mathematics education does not exactly coincide with the content, process and contexts targeted at these levels. The change in the beliefs of teachers and pre-service teachers in mathematics and mathematics education can be effective in developing problems that are appropriate to ML in high-level difficulty.

According to the contexts used, occupational and personal problems were more dominantly posed by the participants in this study. Problems involving scientific and social contexts were determined to be fewer in number. It can be said that the mathematics teachers and pre-service teachers participating in the study have not similar problems concerning the context. In a study by Sahin and Basgul (2018), it was found that pre-service mathematics teachers preferred to develop more mathematical problems in personal and occupational contexts. Concerning context, it is understood that the results in the relevant literature overlap with the results of this research. In addition, it can be said that they have difficulties in developing problems in scientific and social contexts. In the related literature, there are conclusions that pre-service
teachers have difficulty in developing contextual problems. In a related study, Ulusoy and Kepeceoglu (2018) found the pre-service teachers mostly preferred limited number contexts that are familiar, such as money, diet and test-solving in their problems. In another study, Bennison (2015) emphasizes the importance of context for teachers’ development of the ML. The ML should not be taught in the absence of everyday context, and teachers draw from the context which is familiar and unfamiliar to learners (Machaba, 2018). The positive aspects of the ML education given to participants in this study came from the use of various contexts in the problems. However, despite the training provided, it can be said that the contexts in the posed problems are not as effective and qualified as the problems in the PISA study. This can be attributed to that the given training is within a certain period and the participants do not have sufficient knowledge and experience about the context in which they are used. We should provide experience and training to teachers and pre-service teachers in developing different contextual problems. In mathematics teaching and method courses, pre-service teachers can be given theoretical and practical information about contextual learning and context types in which mathematics is used. Activities, such as in-service training, seminars, workshops, are provided to the in-service mathematics teachers about contextual learning.

The problems posed by the participants were found to be more in change-relationships and space-shape concerning mathematical content, but less in the uncertainty-data and quantity content. Teachers posed problems in the most space-figure content, at least in the uncertainty-data content. Pre-service teachers posed more problems in the content of change-relationships, at least in quantity content. It can be said that participants have difficulty in developing problems in uncertainty-data and quantity content. The difficulties of the participants may also arise from the nature of some mathematical concepts. Participants seem to feel more comfortable with developing problems in change-relationships and space-shape content. Pre-service teachers posed more problems in the content of change-relationships, at least in quantity content. It can be said that participants have difficulty in developing problems in uncertainty-data and quantity content. The difficulties of the participants may also arise from the nature of some mathematical concepts. Participants seem to feel more comfortable with developing problems in change-relationships and space-shape content. Sahin and Basgul (2018) showed that the results of this research conducted with pre-service mathematics teachers and the results of this research differ according to the content dimension of the problems. Also, quantity and uncertainty-data contents were more dominant in the problems posed by pre-service mathematics teachers (Sahin & Basgul, 2018). The different results in the studies may be the results of different sample groups and training for ML. A mathematically literate individual can predict, interpret, solve daily life problems, reason in numerical, graphical and geometric situations, and communicate using mathematics (Ojose, 2011). In this context, the interpretation and creation of information with mathematical content in ML should be among the qualifications aimed to be developed in mathematics teaching. In addition, competencies should be aimed to interpret and represent words, forms, symbols, numbers and materials, and mathematical explanations, processes and results (Alsina, 2015; cited in Diaz, 2017). Teachers should integrate their content and/or skills to solve problems and use basic mathematical content for ML (Machaba, 2018). In this study, participants were found to pose more comfortable and effective problems in some mathematics content after ML education. However, in mathematical content, such as uncertainty-data and quantity, fewer and less effective problems posed. This may be due to the beliefs and perceptions of the participants towards ML. It can also be based
on reasons, such as the difficulty and refusal of the participants to experience mathematical content. Mathematics teachers need to develop their skills in problem-posing for ML in different mathematical contents. Mathematics teachers can keep up-to-date and improve their mathematical content knowledge with the help of current journals, books and other related publications.

The problems posed by the participants in this study were higher than the average scores of the skills of employing concerning mathematical processes rather than formulating and interpreting. Mathematics teachers and pre-service teachers have been determined to have a similar situation. Participants seem to have limitations or difficulties in establishing situations involving formulating and interpreting process skills at developing problems. This result may have been influenced by the approach and understanding of problem-based learning and the teaching process in mathematics education. In the related literature, it was found that the problems developed by pre-service mathematics teachers had more productive skills and associative, especially reflective skills, were very low (Sahin & Basgul, 2018). This result can be seen as a direct reflection of the participants’ beliefs about mathematics education because they perceive mathematics as more dominant in the operational dimension.

There were limitations and difficulties in the studies conducted with teachers and pre-service teachers within the scope of ML. In the study conducted by Kabael and Barak (2016), the development of ML of secondary school pre-service mathematics teachers was examined through some PISA questions. Participants were found to be at a disadvantage in mathematization, especially in creating relationships between variables in the problem and in graphical interpretation, while ML was not at the expected level. Lestari et al. (2017) found that many high school mathematics teachers did not have enough knowledge of what ML was and that the exercises included in books for specific topics were in the form of procedural solutions. In addition, many teachers had pessimistic considerations in integrating ML into learning-teaching mathematics. Gurbuz (2014) stated that pre-service teachers were more successful in classifying PISA questions than writing. Moreover, in the study, most of the pre-service teachers reported that they were not aware of the concepts of PISA and ML before teaching. Based on these and similar studies, it is understood that teachers and pre-service teachers have various difficulties concerning theoretical knowledge and application dimensions for ML.

In many countries, the aim of mathematics education is to improve the mathematical competencies of students beyond the operational and conceptual knowledge. Students need to engage in a wide variety of activities, such as reasoning, communicating, connecting, modelling and problem-solving. Teachers should be able to identify their competence needs in activities they want to use in teaching or measuring in mathematical activities (Pettersen & Nortvedt, 2018). In this context, the targeted process skills must be reflected in an effective approach to the problem activities used for the development or measurement of ML. For example, Brown and Schafer (2006) adopted the modeling approaches used as ML tools in their work: formulating, analyzing the model in answering mathematical questions, interpreting
and validating mathematical results concerning context, and reinforcing modeling approach. It was determined that there was a statistically significant difference in the comparison of the mathematical process scores of teachers and pre-service teachers in this research in favor of teachers in the formulation and total scores. It was found that the teachers had higher scores than the pre-service teachers in the total score of process skills. This can be interpreted as that the teachers are one step ahead of the pre-service teachers in reflecting the process skills with problem posing. Although the same ML course is taken, it can be said that this result is due to the richer life experiences of the teachers concerning knowledge, skills and experience than the pre-service teachers.

Ozgen and Kutluca (2013) examined the views of primary school pre-service mathematics teachers on the definition, importance and development of ML. Pre-service teachers had also identified a traditional literacy approach to the definition of ML. Some studies found that pre-service mathematics teachers had limited and incomplete knowledge of the concept of ML (Sefik & Dost, 2016; Yenilmez & Ata, 2013). In the study conducted by Genc and Erbas (2017), high school mathematics teachers stated that the necessity of mathematics education program emphasizing mathematical literacy, content categories, aimed to learn objectives should be defined more clearly. Moreover, in the study conducted by Altun and Akkaya (2014), teachers expressed the reasons for poor achievement in PISA mathematics examinations as a teaching system, examinations, program, teacher and physical environment. Steps should be taken to eliminate these reasons with mathematics educators.

The participants in this study had positive reflections on the problem-posing for ML after the course even though they did not have knowledge, skills and experiences for ML and problem-posing before the ML course. In their study, Sahin and Basgul (2018) stated that most of the problems posed by pre-service mathematics teachers after the training for ML were appropriate to the nature of the PISA study. In this respect, it can be said that the courses or training given to mathematics teachers and pre-service teachers improve their problem-posing skills positively. It can be said that it is especially difficult to develop ML activities based on mathematical problem-solving. Stacey (2005) argued that the important goal of mathematical problem-solving teaching was to understand the problem-solving processes of students, to design excellent tasks and to effective classroom practices. In this context, mathematics teachers’ problem-posing activities for ML and using these problems in their classrooms can be seen as an important goal for mathematics education.

Recommendations

It can be said that the teachers in this study are one step ahead of that they are more successful than the pre-service teachers in posing ML problems. In related studies, it was known that teacher and pre-service teachers had various difficulties or limitations towards ML. Teachers and pre-service teachers’ knowledge, skills and experiences for ML should be provided effectively and adequately. Targeted development can be achieved in training, courses, publications, projects, research and effective learning tools and materials for teachers and pre-service teachers. ML should be seen as a general framework or umbrella in mathematics education. With this approach,
increasing awareness about ML will highly likely to facilitate the targeted development more easily.

It was understood that there was a need for theoretical and practical training that demonstrate the application of ML in different problem types. Challenges to pose unstructured problems for ML should be removed. The situations and content of mathematics used in ML problems should be made more effective and diversified. The importance of teachers’ skills to design different contextual problems should be emphasized. Difficulties in reflecting mathematical processes to ML problems should be eliminated. The importance, necessity and benefits of the ML course have been revealed once again with the results of this study. There were positive reflections on the relationship between ML and the mathematical problem and problem-solving.

This research has limitations in some aspects, such as the lack of pre-assessment, interviews, and reflective thinking writings. In addition, the number of participants in this study and the number of problems posed by the participants can be seen as a limitation. Considering these limitations, more comprehensive studies can be conducted in future studies.

References


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**Matematik Okuryazarlığına Yönelik Problem Kurma Becerileri: Öğretmen ve Öğretmen Adayları Örneği**

Atıf:


**Özet**

Araştırmaın Amacı: Bu araştırmının amacı, matematik öğretmeni ve öğretmen adaylarının matematik okuryazarlığına yönelik problem geliştirme becerilerini incelemektir. Araştırma adayların matematik okuryazarlığına yönelik geliştirdikleri problemlerin türü, zorluk düzeyi, matematiğin kullanıldığı durumlar (bağlam), matematiksel içerik, konular ve süreçler açısından incelenecektir. Ayrıca öğretmen ve öğretmen adaylarının matematik okuryazarlığına yönelik geliştirdikleri problemlerin belirlenen kriterler açısından karşılaştırılması yapılacaktır.


**Anahtar Sözcükler:** Matematik okuryazarlığı, Öğretmen, Öğretmen adayları, Problem Kurma.