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Investigation Of Students' Metacognitive Awareness Failures About Solving Absolute Value Problems in Mathematics Education*

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ABSTRACT

Purpose: Metacognitive awareness plays an important role in assisting students to solve problems because it affects their reasoning in assessing their thinking results. This research aims to investigate students' metacognitive awareness failures about solving absolute value problems in Mathematics Education. **Methodology**: It is qualitative research with 38 subjects of the fifth-semester students of Mathematics Education at the State Islamic University of Maulana Malik Ibrahim Malang. The materials included designing a Metacognitive Awareness Scale, a math assignment sheet related to the absolute value problem and a semi-structured interview sheet.

Findings: It was found that there was a significant failure, and 3 students were sampled from who had experienced different metacognitive awareness failures in solving absolute value problems. The study also proved students' inability to be aware of mistakes in determining problems, algebraic process, and their inability to know that what they carried out was wrong. **Implications for Research and Practice**: This research has implications for teachers, lecturers, and students to utilize the findings of the study in understanding students' metacognitive awareness failures in solving mathematical problems, especially in working on absolute value problems. This is also a reference for teachers on how to teach absolute value problems so that students do not experience metacognitive awareness failure when solving problems.

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Introduction

Metacognition ranks highest in Bloom's revised taxonomy of cognitive tasks as a learning dimension (Karaali, 2015). It refers to a person's ability to know and regulate their cognitive processes (Arends, 2012) for the success of their learning success because it allows learners to manage their cognitive skills and determine their weaknesses as well (Schraw, 1998). Flavell defines metacognition as knowledge of cognitive objects, namely about everything related to cognition (Flavell, 1979). It is defined as a knowledge of strategies used to achieve goals and measure one's progress during or after (Kılınç, 2013). Metacognition is also defined as awareness and regulation of thought processes used by students in planned learning events and problem-solving (Demirel et al., 2015).

These definitions unanimously agree that metacognition is an awareness of what a person understands and what he does not. It is also understood that metacognition is also the knowledge about how to use available information to achieve goals. It is also beneficial in developing cognitive needs and the ability to assess problems. In the field of mathematics, specifically, metacognition plays an important role in problem-solving because it includes awareness of thinking, monitoring, and regulation of cognitive processes (Efklides, 2006; Lingel et al., 2019; Sengul et al., 2012; Wilson et al., 2004). The problem-solving process with the aid of metacognition is highly beneficial for those who have a good knowledge and facts about mathematics; and who have the ability to monitor and manage that knowledge (Miller et al., 2011; Pennequin et al., 2010; Wilson et al., 2004). Hence metacognition helps in developing problem-solving skills as well as cognitive abilities (Goos et al., 2002) to solve mathematics problems.

Zhang et al. (2013) identify two metacognitive components, namely knowledge of cognition and regulation of cognition. Knowledge of cognition is same as awareness and knowledge of cognition itself while regulation of cognition is the ability to control cognitive skills (Elia et al., 2016). The component of awareness is the third major component of metacognition (Abdellah, 2015; Karaali, 2015). Metacognitive awareness component is embedded in students own cognition (Ramirez-Corona et al., 2013) that helps them develop their problem-solving skills (Bars et al., 2017; Hess et al., 2014; Magiera et al., 2011; Mason, 1998).

As a tool for problem solving, metacognition works with four components (Desoete, 2008; Kahwagi-Tarabay, 2010) viz., prediction, planning, monitoring and evaluating. Prediction is the skill that helps in making estimates or predicting something; planning is the skill of designing solutions; monitoring refers to a person's awareness which is in line with understanding and carrying out tasks; and evaluating is the skill of assessing the solutions and process of one's learning settings. Metacognitive awareness has also been associated with various indicators of solving problems (Magiera et al., 2011; Wilson et al., 2004) such as (1) knowing what is known from a given mathematical problem; (2) relating the problem to a previous problem; (3) comparing the problem with a previous problem; (4) knowing what to do; and (5) knowing what has been done. These indicators of metacognitive awareness can be used for solving math problems.

Research also suggests that metacognition related to problem-solving is a complex cognitive task (Lucangeli et al., 1997) but has received very little attention (Ackerman et al., 2017). It is also suggested that metacognitive awareness processes vary for each cognitive tasks and that the cognitive processes are stronger in learners having superior cognitive skills and less in those who experience metacognitive failures (Livingston, 2003), though the relationship between metacognition and intelligence is yet to be empirically proven (Stankov, 2000). There is however no dearth of research on metacognitive failure such as students experiencing metacognitive failure when solving math problems (Goos, 2002); or when they complete a mathematical proof process (Huda, 2016) or while solving problems in the field of statistics (Rozak, 2020) or when studying lesson content (Yüksel et al., 2012). Most of these metacognitive failures are a component of metacognitive awareness, especially in receiving information (Purnomo et al., 2017). Such a wide range of experiences of metacognitive failure, especially in the awareness component, necessitates a deeper study to understand the causes and impact of failures of students' metacognitive awareness.

It was therefore important to examine how learners can cope up with failures. The above mentioned definitions and discussion on metacognition suggest that there are at least three components in metacognitive abilities: first, it is awareness which is a person's basic ability to reflect an understanding of the problem situation and then assume it; second, there is a regulation of cognitive activities which helps students choose a goal, a strategy, and then implement it by developing a relationship with their ability to manage abilities with mathematical results; finally, cognitive evaluation as an activity that requires students to act as problem solvers and explicitly reflect on what will be done during the problem-solving process.

So far studies on metacognitive awareness have discussed problem solving skills in the fields of integral material, statistics, and linear equations. But in this study, the researcher attempted to discuss the issue of absolute value. The main objective was to investigate students' metacognitive awareness failures about solving-problems related to absolute value in Mathematics Education. Absolute value is one of the most difficult materials for students to solve (Almog et al., 2012; Amram, 2019; Elia et al., 2016; Park et al., 2019). Learners have several errors in solving absolute value problems. One of them is misinformation or lack of information and awareness (Almog et al., 2012). This misinformation or lack of awareness results in students' experience of failures since errors in learning and solving absolute value problems is a wrong strategy (Elia et al., 2016) which often demotivates students to find the right information.

The aim of this study was therefore to attempt a new metacognitive awareness scale for university undergraduates based on the obstacles and failures. The tool developed should reflect their cognitive skills as well as their metacognitive awareness about absolute value problems. This domain-specific scale should also help measure participants' awareness of their own metacognition failures. However, the main instrument in this research would be the researcher. The supporting instruments were the metacognitive awareness scale and student assignment sheets given to participants to solve mathematical problems and observation sheets to confirm the failure of student awareness in solving absolute value problems. The assignment to students in this study consisted of one specific problem developed by the researcher. The specific problem here was a problem that did not need any procedural resolution. The solution could be with conceptual knowledge alone. Thus, the completion of the task required students' metacognitive awareness. This research can serve as an important source of information for teaching staff who are willing to identify the causes of the failure of metacognitive awareness. It will add a new dimension in the metacognitive awareness failures experienced by students in solving the problems of absolute value.

Method

Research Design

This study used a descriptive exploratory method with a mixed research design, i.e., with both quantitative and qualitative approach. The mixed method design was appropriate to understand the failure of students' metacognitive awareness in solving absolute value problems. While the quantitative part enabled a descriptive statistical investigation, the qualitative part carried out the content analysis of the students' assignments. The present study was conducted in sequential mixed methods design in which the collection of quantitative data was followed by the collection of qualitative data (Klassen et al., 2012). According to Creswell (2003), mixed research design eliminates the complexity of a research.

Research Sample

The sampling of the study consisted of 38 students selected through stratified random sampling method from the fifth-semester of Mathematics Education at the State Islamic University of Maulana Malik Ibrahim Malang, considering their gender and academic levels.

Data Collection Instruments and Procedures

The research was conducted at two levels: at the quantitative level, the participants were given the 19-item Metacognitive Awareness Scale, which was designed for the quantitative phase of data collection and its analysis involved a discriminatory index and exploratory factor analysis. The validity and reliability of the scale was determined prior to administering it. These findings were integrated and interpreted to arrive at conclusions (Berman, 2017). This step was followed by a semi-structured interview with three selected participants who had faced major problems in determining the form of equality and inequality of variables and could not understand the meaning of inequality.

Data Analysis

The data analysis aimed at examining the relationships between the variables of Metacognitive Awareness and ability to solve Absolute Value Problems of students. The sampled three students were given an assignment to solve by using the absolute value problems. They were randomly picked from those who had failed to solve problems. These students showed different types of metacognition awareness failure in solving absolute value problems. These three students were the focus of the qualitative phase of this study The content analysis and verbatim remarks of these students (S1, S2 and S3) provided a first-hand account of the failure of metacognitive awareness. Their written assignments also became the indicators of the level of metacognitive awareness. This was also the reflection of their lack of self-awareness, attention potential, and recall of knowledge. This constituted the qualitative phase of the study. The interview focused mainly on whether the metacognitive awareness scale could measure their psychometric criteria and various forms of metacognitive components

The following example illustrates the failure of metacognitive awareness in solving a problem of absolute value. The students were given the task:

Determine the solution of the following inequalities!

$$|x-1| + x^2 - 6x + 9 \geq 0$$

After determining the participants, the researcher interviewed students based on the results of their written answers. Semi-structured interviews were used to explore the causes of the failure of metacognitive awareness of students in solving absolute value problems. Interview guidelines were based on indicators of metacognitive awareness in solving problems. The assignment sheet and observation sheet received validation from three experts (mathematics education) before the researchers distributed questions to students. The validation material was adjusted to meet the suitability of the subject matter which can reveal the student's awareness in solving mathematical problems.

Results

Keeping the objective of this research in mind, a Metacognitive Awareness Scale was adapted from a Metacognitive Awareness Inventory (Schraw, 1998) as a short pilot version to evaluate metacognitive Awareness of the sample of this study. It consisted of 19 items divided into four domains: Attention (with 4 items), Recall of knowledge (5 items), Metacognitive awareness (6 items), and Cognitive skills (4 items), and utilized a 5-point Likert-type scale. These 19 items represented the mathematical problems usually faced by students. After administering the scale, the first step was to validate the construct of the scale Table 1 presents Pearson correlation coefficient of the items with all its four domains.

Table 1

Pearson Correlation Coefficient of The Items

Domain	Number of items	Correlation of the items with their domain	Correlation of the items with the whole scale
Attention	4	0.303-0.527	0.350–0.744
Recall Of Knowledge	5	0.357–0.687	0.421–0.528
Metacognitive Awareness	6	0.312–0.672	0.489–0.633
Cognitive Skills	4	0.344–0.580	0.477–0.766

* All the values were statistically significant at (α=0.05).

Intercorrelation coefficients of the items within their domain using the Pearson correlation coefficient ranged between 0.303–0.580. The correlation of the items was also calculated with the whole scale and the score range fell between 0.350–0.766. All values were statistically significant at α = 0.05, which shows the scale had good construct validity.

Both the correlation coefficients were measured to calculate Cronbach's coefficient alpha and internal consistency coefficient of the items. Table 2 shows the internal consistency coefficient according to Cronbach's alpha formula and the transferability and reliability of all items within the scale. The Cronbach's alpha reliability coefficient normally ranges between 0 and 1; the closer the coefficient is to 1.0, the greater is the internal consistency of items in the scale. Hence, this would mean that Cronbach's alpha coefficient can be increased if the number of items increase,

Table 2

Internal Consistency Coefficient, Alpha Cronbach and Transferability and Reliability of All Items.

Domain	Number of items	transferability and reliability	Internal consistency	Cronbach's alpha	Composite Reliability
Attention	4	0.88	0.85	0.75	.643
Recall Of Knowledge	5	0.90	0.91	0.73	.689
Metacognitive Awareness	6	0.89	0.89	0.89	.616
Cognitive Skills	4	0.90	0.95	0.81	.676

The internal consistency composite (construct) reliability, transferability and reliability and Cronbach alpha and were found satisfactory. The internal consistency for each domain exceeds 0.7 (Cronbach, 1951) while composite reliability should be greater than at least .60 (Fornell & Larcher, 1981).

Table 3 presents descriptive analysis of all 19 items calculated to test item reaction and discrimination indices. The convergent validity was also determined to test the degree of agreement between items of the same construct (Ab Hamid et al., 2017). Table 4 presents the value of AVE for each domain as satisfactory since it is higher than 0.5, required to verify the convergent validity (Fornell & Larcker, 1981). Likewise, discriminant validity of a scale is required to calculate to ensure that variance is due to the latent variable and not due to any measurement error or any external effect (Fornell & Larcker, 1981). In order to verify discriminant validity, the AVE estimates of all four domains were compared with the square of correlation (shared variance) among the factors (Hair et al., 2010). Table 5 shows AVE values in bold that are higher than the row and column values which indicates the shared variance among the domains.

Table 3

Descriptive Statistics

Items	Mean	SD	Skewness	SE	Kurtosis	SE	Remarks
1.	4.12	1.23	-0.341	0.001	-0.341	0.101	Agree
2.	4.29	1.34	-0.341	0.001	-0.341	0.101	Agree
3.	4.11	1.23	-0.341	0.001	-0.341	0.101	Agree
4.	4.21	1.33	-0.341	0.001	-0.341	0.101	Agree
5.	4.23	1.30	-0.341	0.001	-0.341	0.101	Agree
6.	4.12	1.23	-0.341	0.001	-0.341	0.101	Agree
7.	4.09	1.20	-0.341	0.001	-0.341	0.101	Agree
8.	4.11	1.23	-0.341	0.001	-0.341	0.101	Agree
9.	4.23	1.33	-0.341	0.001	-0.341	0.101	Agree
10.	4.34	1.43	-0.341	0.001	-0.341	0.101	Agree
11.	4.12	1.23	-0.341	0.001	-0.341	0.101	Agree
12.	4.23	1.34	-0.341	0.001	-0.341	0.101	Agree
13.	4.20	1.33	-0.341	0.001	-0.341	0.101	Agree
14.	4.11	1.23	-0.341	0.001	-0.341	0.101	Agree
15.	4.28	1.33	-0.341	0.001	-0.341	0.101	Agree
16.	4.22	1.33	-0.341	0.001	-0.341	0.101	Agree
17.	4.77	1.53	-0.341	0.001	-0.341	0.101	Agree
18.	4.41	1.43	-0.341	0.001	-0.341	0.101	Agree
19.	4.11	1.23	-0.341	0.001	-0.341	0.101	Agree

*SE = standard error

Table 4

Convergent Validity

Domain	Items	AVE
Attention	4	.534
Recall Of Knowledge	5	.512
Metacognitive Awareness	6	.556
Cognitive Skills	4	.532

The Metacognitive Awareness Scale was administered on all 38 students of the study. Results in Table 6 show that Attention domain scored (M = 52.64, SD = 3.78); Recall of knowledge scored (M = 51.69, SD = 5.47), Metacognitive Awareness scored (M = 51.69, SD = 5.47), and Cognitive Skills scored (M = 51.69, SD = 5.47), which are significantly higher mean scores.

Finally, the Kolmogorov-Smirnov and Shapiro-Wilk tests were performed which allowed a comparison of data distribution with the normal standard distribution

(Riadi, 2016). Table 8 illustrates that the data was normally distributed, which is further evident in the large deviation having a low p-value < 0.05. If p < 0.05, variables follow a normal distribution in the domains.

Table 5

The Discriminant Validity Index Summary

Domain	Attention	Recall Of Knowledge	Metacognitive Awareness	Cognitive Skills
Attention	**0.550**			
Recall Of Knowledge	0.168	**0.522**		
Metacognitive Awareness	0.102	0.423	**0.608**	
Cognitive Skills	0.100	0.345	0.545	**0.568**

Table 6

Mean and SD Across Domains

Domain	Items	Mean	SD	T	Df	Sig
Attention	4	17.64	4.78	1.84	595	.016
Recall Of Knowledge	5	21.69	5.47	1.78	675	.011
Metacognitive Awareness	6	25.12	7.94	1.98	650	.022
Cognitive Skills	4	17.64	4.75	1.00	550	.055

Analysis of variance in Table 7 showed a significant main effect on all items, suggesting a high and significant variance in all domains. ANOVA test determines the mean difference between several domains using analysis of variance (Riadi, 2016).

Table 7

ANOVA Test Results of All Items

Domain	Items	Sum of Squares	Df	Mean Square	F	Sig.	Partial Eta Squared
Attention	4	62.46	595	27.16	2.40	.016	.039
Recall Of Knowledge	5	83.24	675	25.94	3.50	.011	.056
Metacognitive Awareness	6	90.40	650	26.13	4.80	.022	.069
Cognitive Skills	4	67.17	550	27.28	2.40	.055	.039

The Qualitative Data

Out of the 38 students who solved the problem, three (3) students experienced different metacognitive awareness failures in solving the problem of absolute value.

Table 8

Normality Test Results of Scores Related to Metacognitive Awareness Scale

	Kolmogorov-Smirnov			Shapiro-Wilk		
Domain	Statistics	N	P	Statistics	N	p
Attention	.202	38	.000	.897	38	.001
Recall Of Knowledge	.120	38	.007	.945	38	.007
Metacognitive Awareness	.309	38	.000	.621	38	.000
Cognitive Skills	.250	38	.000	.815	38	.000

First Subject (S1)

S1 commenced his attempt by changing the shape $x^2 - 6x + 9$ to be $(x-3)^2$. Then S1 changes the form of absolute values into quadratic forms. This made S1 describe inequality more simply. Hence, S1 was combining one tribe with another. However, the acquisition of new inequality made it even simpler. Consequently, S1 found it difficult to find a solution. Finally, S1 did not get the solution as expected. Figure 2 illustrates the results of the S1 answer and Figure 3 records researchers remarks on it.

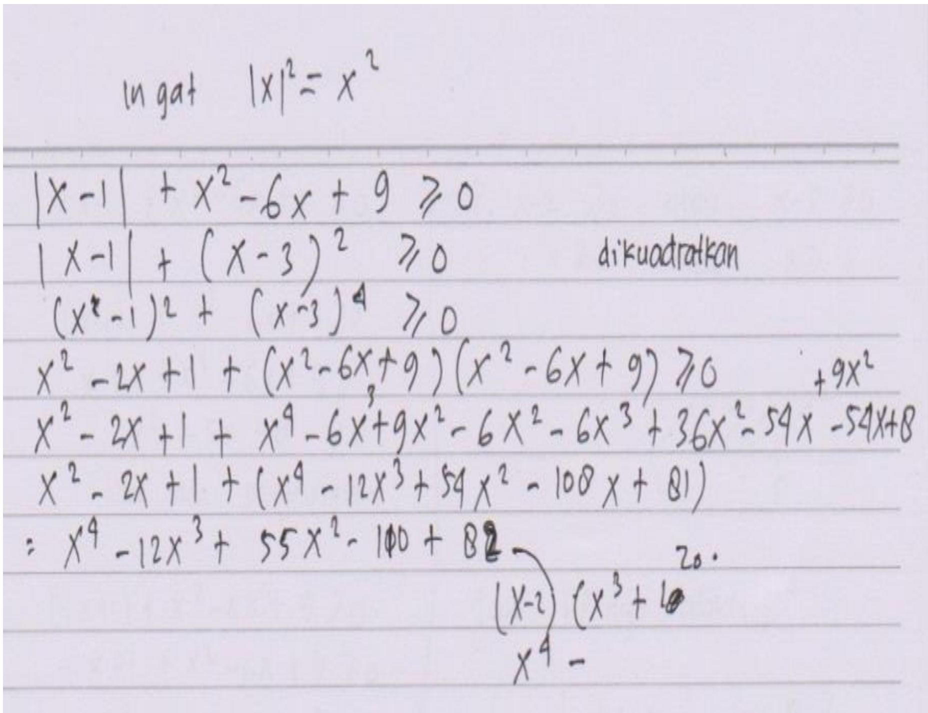

Figure 2. Results S1

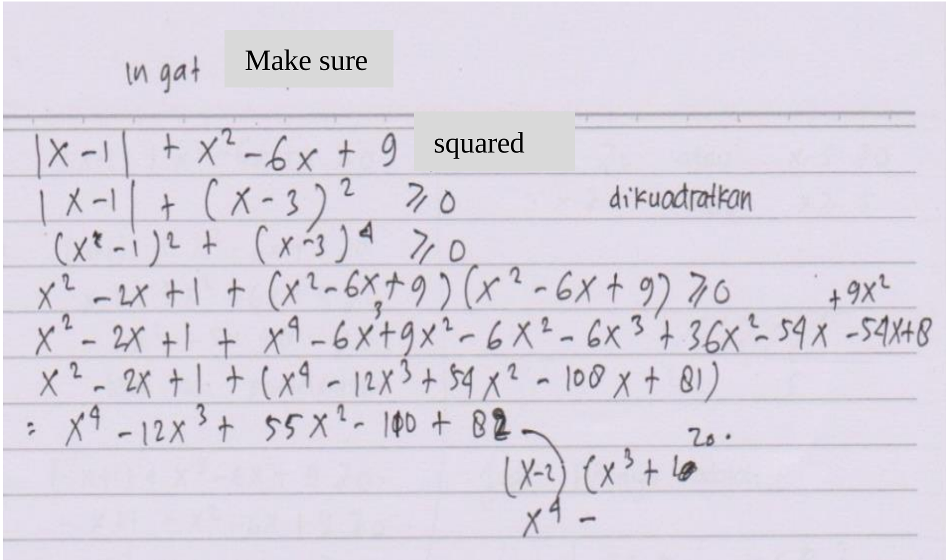

Figure 3. *Observation Sheet of Figure 2.*

Researcher then interviewed S1 to dig deeper. The following excerpt was the result the semi-structured interview with S1:

Q. : "What do you know about problems?"
S1. : "The equation contains absolute values, and we find the value of x that satisfies the problem."
Q. : "Is there information that you can find in the problem?"
S1. : "Hm, the equation contains absolute values."
Q. : "What is the use of squaring in the form of absolute value squared?"
S1. : "To simplify the equation so that it is easier to find the value of x."
Q. : "After squaring, what do you get?"
S1. : "Maybe because I was not careful enough to simplify, finally I did not get the expected x value."

From the results of the interview, S1 assumes that S1 cannot solve the problem because he is not thorough in the algebra process.

Second Subject (S2)

S2 was able to solve the problem by separating the equation into 2 parts based on the concept of absolute value, namely inequality $(x-1)+x^2-6x+9=0$ and $-(x-1)+x^2-6x+9>0$. Regarding the first inequality, S2 stated that there was no solution. While from the second inequality S2 got a new inequality, that is $(x-2)+(x-5)\geq 0$. However, S2 made a mistake in the algebra process. There was no conformity with the results and S2 was not aware of the error. S2 also did not check the values. Therefore, S2 immediately concluded the results whatever he obtained. Figure 4 presents the results of S2's attempt and the researcher's remarks are in Figure 5.

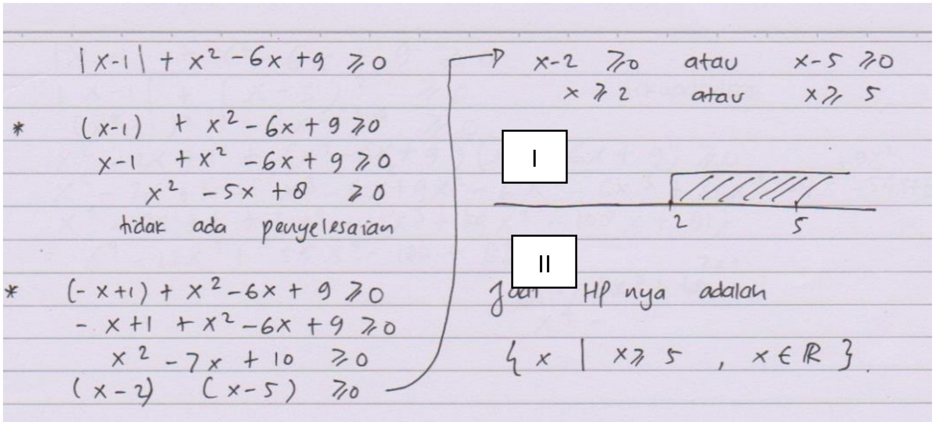

Figure 4. *Results S2*

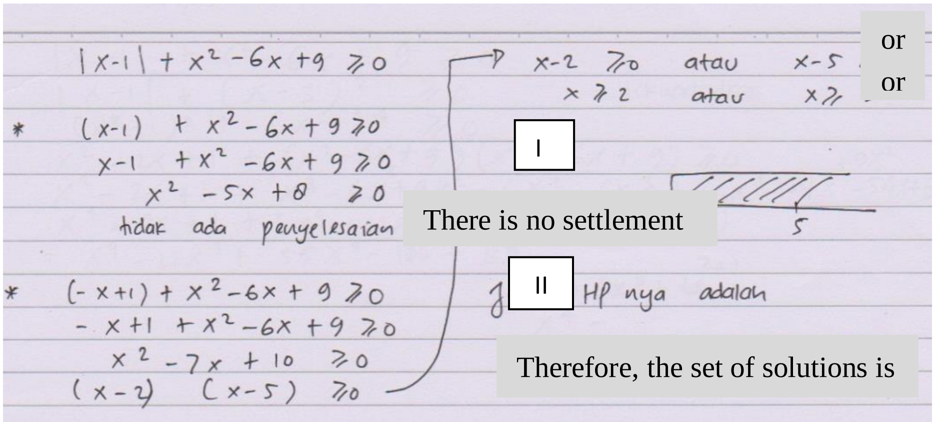

Figure 5. *Observation Sheet of Figure 4.*

When the researcher interviewed S2, the following was the outcome:

Q. : "What do you usually do to solve problems like this?"

S2. : "There was an absolute value; I divided the absolute value into two parts as in this problem that is $(x-1)$ and $-(x-1)$".

Q. : "What did you get?"

S2. :" From the I equation, I tried to get the value that would satisfy but it did not exist, because the roots of the equation were imaginary numbers. Whereas from equation II, I got a value $x \geq 5$"

Q. : "Are you sure about your answer?"

S2. : "I am sure because I have checked several values $x \geq 5$, and the results meet the equation."

S2 did not realize the roots of the equation. The value satisfied the square of inequality. The result in S2 was wrong in stating the results of inequality. The researchers tried to invite S2 to check the value other than $x \geq 5$. S2 only realized that the result of his work was still wrong. The researchers tried to invite S2 to look again at the inequality in the problem carefully. After S2 looked at the inequality of the questions for about 5 minutes, S2 realized that the real results were all $x \epsilon R$. Then the researchers asked about the obstacles in solving the problems. He said that he immediately worked on the problem in the usual ways as he used to practice. He did not see complete inequality. He only saw the numbers without interpreting the numbers. He said that he had seen the ≥ 0 marks in the account, but he did not interpret the sign.

Third Subject (S3)

S3 solved the problem with the same strategy as that of S2. Therefore, S3 too could not conclude the results he obtained. Judging from the results of the answers, S3 was almost able to solve the problem, but S3 could conclude the results obtained. The results of S3's answer are in Figure 6 and the researcher's remarks in Figure 7.

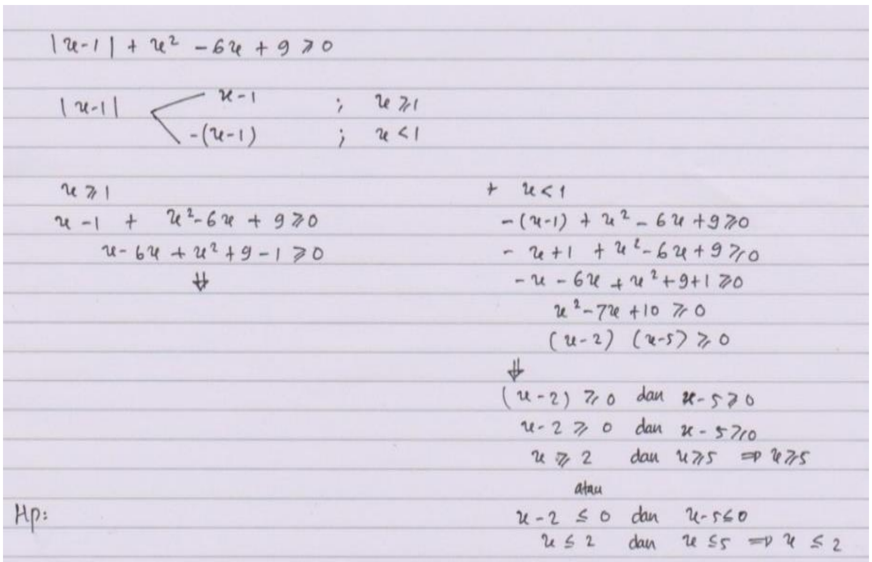

Figure 6. *Results S3*

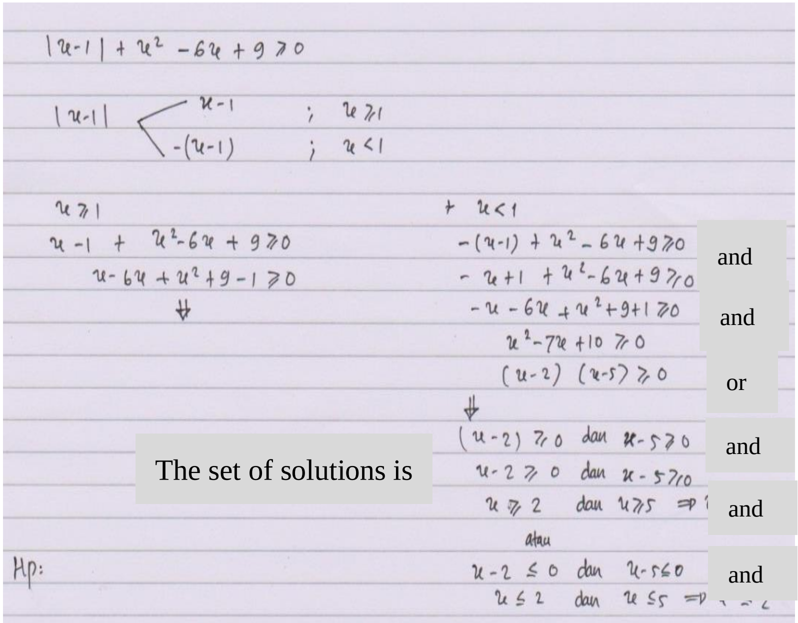

Figure 7. *Observation Sheet of Figure 6.*

Based on the results of the interviews, it was obvious that S3 experienced confusion in concluding the results. The following excerpts from interviews with S3 prove this.

Q. : "Why haven't you concluded the results?"

S3 : "I was confused. The result that I got seemed was not quite right."

Q. : "Why?"

S3. : "I checked the results I got, the result met the equation, I also checked the value of 3, but the result also met the equation." “I became confused."

Based on the results above, S3 realized that the result of his work was still wrong. However, S3 did not realize where his mistake was.

The results of the written answers and interviews of all three participants (S1, S2, and S3) however do not hint directly at what the problem is. It is evident that S1, S2, and S3 did not realize that they had not fully understood the problem. S1 was not able to pull all the information in the problem. S1 only looked for values x that satisfied the equation. Whereas both S2 and S3 felt that the equation contained absolute values and the equation was greater than zero. But S2 and S3 could not interpret that information. Therefore, both S2 and S3 could not apply it in the problem-solving process. They did not realize that the problem was a matter of inequality of absolute value. However, based on the results of interviews, S1 and S2 still felt that the questions were similar. Procedurally, S2 and S3 were appropriate in designing strategies. However, S2 and S3 mad mistakes in the algebraic process. Consequently, their solutions did not match the expectations. S2 did not realize his mistake but S3 did realize and admitted that the results of his work were incorrect though he did not find that error.

Figure 8 identifies the mistakes and shows how S2 was wrong in solving problems procedurally. One of S2's mistakes was that he was unaware of the decision-making in providing solutions to inequality I. From the results of S2's decision in inequality I, S2 considered that inequality I was an equation.

These findings are consistent with a few studies like, (El-khateeb, 2016; Rowntree, 2009) which also observed students experiencing obstacles in shaping the questions and in determining the form of equality and inequality of a variable. This finding is echoed in the experience of S1 and S2, who were not able to fully understand the meaning of inequality. S1 and S2 lacked awareness of the important information in the matter. S1 and S2 did not have the characteristics of self-awareness, namely attention, and recall of knowledge.

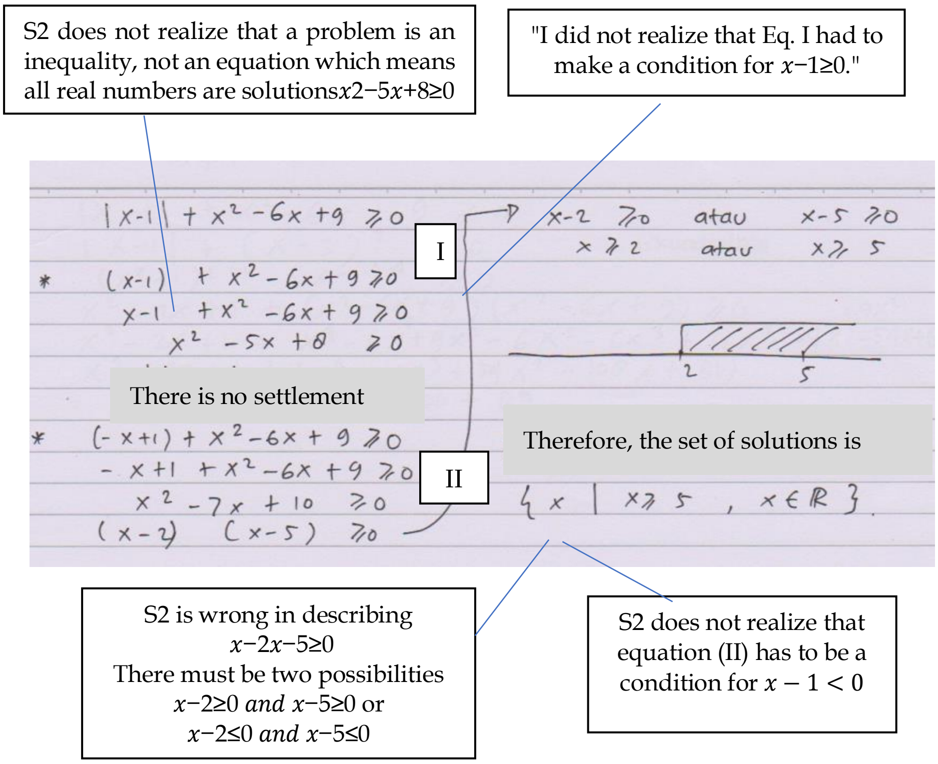

Figure 8. *The Failure of S2 Awareness in Solving Absolute Value Problems*

Discussion, Conclusion and Recommendations

A study (Solso, 2008) illustrated that characteristics of consciousness namely attention, wakefulness, architecture, recall of knowledge, and selectivity are required by students to solve practical problems without requiring procedural completion. In our study, too, S1 and S2 were not aware of these characteristics. They could not fully understand the problem and the concepts of absolute values, squares, and inequality nor could directly answer the solution of the existing problems. In general, students always worked procedurally.

(As' ari et al., 2017) stated students were accustomed to do things of a procedural nature. So, they were not aware of what the real problem was. These findings are consistent with a few studies (El-khateeb, 2016; Rowntree, 2009) which also observed students experiencing obstacles in shaping the questions and in determining the form of equality and inequality of a variable. This finding is echoed in the experience of S1 and S2, who were not able to fully understand the meaning of inequality. S1 and S2 lacked awareness of the important information in the matter. S1 and S2 did not have the characteristics of self-awareness, namely attention, and recall of knowledge. Likewise, a study (Solso, 2008) illustrated that characteristics of consciousness namely attention, wakefulness, architecture, recall of knowledge, and selectivity are required by students to solve practical problems without requiring procedural completion. In our study, too, S1 and S2 were not aware of these characteristics. They could not fully understand the problem and the concepts of absolute values, squares, and inequality nor could directly answer the solution of the existing problems. In general, students always worked procedurally. (As' ari et al., 2017) stated students were accustomed to do things of a procedural nature. So, they were not aware of what the real problem was.

If we ponder upon the outcomes of the failure for each of the three students, we understand that S1 failed to realize many things about the problem, though he knew them: namely he failed to realize the information in question; he failed to realize that he was wrong in determining the problem; he also failed to realize the algebraic process which he had done was wrong. The failure of S1's awareness was also due to the lack of motivation in him to look further when he was unable to solve the problem. (Karaali, 2015) believed that strong motivation was very helpful for students to raise students' metacognitive awareness. The lack of motivation of undergraduate students results in the situation what was faced by S1, who failed to find solutions again what he had not yet obtained.

S2 and S3 solved the problem by separating the equation into two parts based on the concept of absolute value. This shows that S2 and S3 used their knowledge when there were new problems. Piaget (Ultanir, 2006) argues that a child brings new knowledge into its scheme. Although S2 made several mistakes such as thinking that the problem he was working on was a matter of equality. This is consistent with El-khateeb (2016) and Rowntree (2009) who stated that students made mistakes in solving the problem of absolute value, which often assumed the form of inequality of absolute value.

Almog et al. (2012) also stated that students experienced misconceptions about the use of signs of inequality absolute values. This justifies the error experienced by S2. However, S2 did not realize that error nor did he check the values of the solutions. The researcher tried asking S2 to check some values from the solution but S2 realized that he had made a mistake in solving problems. Based on the above, the failure of metacognitive awareness S2 is also a failure to know the information about the problem and a failure to realize that the algebra process has errors. The failure of this metacognitive awareness is influential because S2 is not focusing on seeing the problem. Thus, S2 happens to be a case of lack of the concept of prior knowledge.

On the other hand, the mistake committed by S3 was of different nature. S3 did not include a condition of absolute value so he was wrong in making decisions. S3 also experienced confusion in deducing inequality results of I, which was confirmed by the results of the interview. This proved that S3 was lacking the mastering skills of inequality. After checking the results, S3 realized that the results of his work were still not quite right. However, he did not find out what his mistake was. This means that S3 was not aware of the algebraic process. He experienced the failure of metacognitive awareness as he did not realize that there was information on questions that he could not yet use.

Such failure are evidences of students' metacognitive awareness and are associated with students' metacognitive skills (Desoete, 2008; Kahwagi-Tarabay, 2010). These evidences prove that metacognitive skills were still low which was because each of the three sampled students had committed mistakes. If S1 had not been able to predict, plan, monitor, and evaluate, S2 and S3 were also not aware of the processes, which also amount to low metacognitive skills.

This study proved four findings. First, students fail to realize information about questions; second, they fail to realize mistakes in determining problems; third, they lack the awareness of the algebraic processes; and finally, they fail to identify what wrong they had done. Though students admit that that they cannot focus on seeing the real problems; a few believed that they knew that there was something important in the problem. Even though the problem would occur, they would not be able to identify it and solve it. Developing students' logical reasoning skills plays an important role in teaching activities. It may be useful for teachers to structure their lessons according to the cognitive awareness of students. This research recommends conducting further research on the level of students' metacognitive awareness in solving absolute value problems. Research can also be carried out on other domains of metacognitive awareness and on a larger sample.

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